

DUAL-STATE KALMAN FILTER FORECASTING AND CONTROL THEORY
APPLICATIONS FOR PROACTIVE RAMP METERING

by

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DEDICATION

“Forgetting what is behind and straining toward what is ahead, I press on toward the goal...” (Philippians 3:13-14)

The work herein is a true labor of love, but the “ah-ha” moments did not arrive without toil and occasional bouts of weariness. With its completion, I hope any door I’ve been given the opportunity to walk through is wider for any who may follow. Throughout this pursuit, many people have extended encouragement and support that has contributed to this thesis in ways that are immeasurable. I would personally like to thank my family and friends, especially the Gould family, whose kindness has altered the course of many lives, the Robinson family, and Scott Reynolds, whose friendship is unassailable.

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AUTOBIOGRAPHICAL SKETCH OF AUTHOR

Brian Portugais received his B.S. in Civil Engineering from Boise State University in 2012. As an undergraduate, he worked as a research assistant modeling the flow of groundwater. His senior design project, “Ten Mile Road Interchange and Bridge Design” won “Best Poster” from the Department of Civil Engineering and shifted his focus to transportation. As a Graduate Research Assistant, Portugais has published two peer-reviewed journal articles with his advisor, Dr. Mandar Khanal.

ABSTRACT

Deterioration of freeway traffic flow condition due to bottlenecks can be ameliorated with ramp metering. A challenge in ramp metering is that it is not possible to process data in real-time and use the output in a control algorithm. This is due to the fact that by the time processing is completed and a control measure applied, the traffic state will have changed. A solution to this problem is to forecast the traffic state and implement a control measure based on the forecast.

A dual-state Kalman filter was used to forecast traffic data at two locations on a freeway (I-84). A Kalman filter is an optimal recursive data processing algorithm; predictions are based on only the previous time-step's prediction and all previous data do not need to be stored and reprocessed with new measurements. A coordinated feedback ramp metering control logic was implemented. The closed-loop system seeks to control the traffic density on the mainline while minimizing on-ramp queues through weighting functions.

The integration of the Kalman filter with the ramp meter control logic accomplishes the ramp meter algorithmic scheme, which is proactive to changes in freeway conditions by controlling a forecasted state. In this closed-loop framework, real-time forecasts are produced with a continuously updated prediction that minimizes errors and recursively improves with each successive measurement. MATLAB was used to

model the closed-loop control system as well as modify the input output constraints to evaluate and tune controller performance.

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LIST OF NOMENCLATURE

Bold variables indicate vector notation

\mathbf{x}_k	State vector
\mathbf{y}_k	Observation vector
\mathbf{G}_k	Known state matrix
\mathbf{F}_k	Design matrix of known values of independent variables
\mathbf{w}_k	State process noise assumed to be Gaussian white noise with $\mathbf{w}_k \sim \mathcal{N}(0, \mathbf{Q}_k)$
\mathbf{v}_k	Observation noise assumed to be Gaussian white noise with $\mathbf{v}_k \sim \mathcal{N}(0, \mathbf{R}_k)$
\mathbf{K}_k	Kalman filter gain
$\hat{\mathbf{x}}_0$	Initial state estimate
\mathbf{P}_0	Initial error covariance estimate
$\hat{\mathbf{x}}_{k k-1}$	State estimate <i>a priori</i>
$\hat{\mathbf{P}}_{k k-1}$	Error covariance estimate <i>a priori</i>
$\tilde{\mathbf{y}}_k$	Measurement innovation
$\hat{\mathbf{x}}_{k k}$	State estimate <i>a posteriori</i>
$\hat{\mathbf{P}}_{k k}$	Error covariance estimate <i>a posteriori</i>

v_{f1}	Free flow speed of section 1
v_{f2}	Free flow speed of section 2
ρ_{om1}	Jam density of section 1
ρ_{om2}	Jam density of section 2
ρ_{oc1}	Critical density of section 1
ρ_{oc2}	Critical density of section 2
Δx_1	Section 1 length
Δx_2	Section 2 length
ρ_1	Section 1 density
ρ_2	Section 2 density
r_1	Ramp 1 demand
r_2	Ramp 2 demand
u_1	Ramp 1 metered flow
u_2	Ramp 2 metered flow
l_1	Ramp 1 queue length
l_2	Ramp 2 queue length

CHAPTER 1: INTRODUCTION

Traffic congestion is often an adverse effect of population increase and economic expansion. As the Treasure Valley experiences rapid growth, congestion management will be vital as there is an increase in demand for highway travel and vehicle miles traveled. An increase in capacity can result in congestion levels that quickly become similar to those prior to adding the additional capacity and it is unlikely new construction will ever catch up due to funding and land use limitations. Since new construction is often the last resort to improve the operations of a freeway, strategies that are more cost-effective and utilize existing infrastructure or require minimal expansion are needed to alleviate congestion in the region.

1.1 Background

Interstate-84 (I-84) and Interstate-184 (the Connector), a freeway linking downtown Boise with I-84, are the backbones of the Treasure (Boise) Valley's transportation system. I-84 and I-184 are the primary connections between the region's major employment, activity, and retail centers. Current weekday traffic volumes on I-84 range from 20,000 north of Canyon County to 120,000 between the Eagle Road and Wye Interchanges in Ada County. By 2035, it is forecast that the travel demands on this corridor will double (COMPASS, 2013).

The 2010 census reported the population of Boise and the metropolitan area were 205,671 and 616,561, respectively (US Census, 2012). This is an increase in the

metropolitan area by almost 138,000 since 2002 (Idaho Department of Labor, 2013). The metropolitan area, consisting of Ada and Canyon Counties, is home to about 600,000 people and The Community Planning Association of Southwest Idaho (COMPASS) forecasts by 2040 Ada and Canyon Counties will have combined a population of 1,022,000 and 462,000 jobs (COMPASS, 2014). The existing delay on the average weekday is 27,670 hours (COMPASS, 2014).

In the U.S., traffic congestion cost drivers more than \$ 100 billion in 2011 (Schrank, Eisele, & Lomax, 2012). Nonrecurring congestion predominantly results from incidents (accidents or breakdowns), work zones, and weather. Recurring congestion most often occurs routinely during peak hours and is simply the result of traffic demand exceeding freeway capacity usually at a bottleneck. A freeway bottleneck is a critical point on the road characterized by freely flowing traffic downstream with queues upstream. This can occur wherever there is a constriction of capacity or demand exceeds capacity. Hidden bottlenecks exist at locations where the demand exceeds the capacity, but the demand cannot reach the hidden bottleneck location because of the presence of upstream or downstream primary bottlenecks. A micro-bottleneck is identified by perturbations from slow-and-go wave oscillations and a downstream critical bottleneck. Geometric features that contribute to the occurrence of freeway bottlenecks include:

- On-ramp sections with no auxiliary lane additions, or with short acceleration lanes
- Weaving sections, particularly out of dropped lanes
- Lane drops on basic segments, or following an off-ramp
- Long upgrades, particularly in the presence of heavy vehicles

- Narrowing lane conditions
- Lateral obstructions which reduce free flow speed (FFS), particularly on bridge sections

I-84 experiences recurring and nonrecurring congestion resulting from bottleneck formations, especially between Eagle Road and Meridian Road interchanges. By 2040, the average weekday delay will increase by a forecasted 1500 percent to approximately 440,980 hours (COMPASS, 2014). Utilizing technology and innovative solutions are crucial to accommodate the forecasted growth and manage its certain congestion. One such solution is ramp metering, which was reported to have saved \$1.8 billion in congestion costs from 1982 to 2002 (Schrank & Lomax, 2004).

1.2 Problem Statement

Currently, ramp metering is not used in Idaho. In this thesis study, we investigate new ramp metering to alleviate congestion. The methodology combines the use of Kalman filters for short-term forecasting of traffic variables and the use of feedback control theory to develop the ramp metering scheme. The need to forecast traffic variables with an adaptive tuner arose from a challenge in processing real-time traffic data. Many ramp metering schemes attempt to maximize throughput by metering to a predetermined optimal occupancy or maximum capacity level. This suffers from the fact that capacity is known to not be a fixed value and the optimal measure of it may change under a wide range of conditions. The method proposed in this study is a stochastic modeling approach that is adaptive to conditions (e.g., driver behaviors, adverse weather, incidents, and etc.) and is applied in an on-line manner, yielding real-time forecasts.

The Eagle interchange currently contributes the most vehicles to I-84 and is within two miles of the Meridian Road interchange. The Eagle Road East Bound (EB) loop on-ramp enters vehicles on the mainline 2,400 feet upstream of the merge of the Eagle Road EB on-ramp. Ramp metering at the EB on-ramp has been shown through simulation to improve flow entering the mainline, allowing throughput speeds to be maintained, eliminating potential shockwave effects due to the on-ramps close proximity of one another. With the recent construction of the new interchange at Ten Mile Road and the planned construction of the Meridian Road interchange, system-wide improvements in travel time and reduction in delays could result with ramp meters at these locations.

These areas highlight the significance for a ramp metering system that is locally adaptive to the on-ramp conditions and dynamically coordinated between the interchanges, to effectively manage the overall flow through the corridor. However, few reliable automatic control strategies exist due to the complexity of the traffic flow phenomenon (Adeli & Karim, 2005). Though a number of coordinated traffic-responsive strategies have been proposed, few have been implemented due to the computational time required for the algorithms (Gokasar, Ozbay, & Kachroo, 2013).

1.3 Thesis Summary

Based on these relationships, a predictive feedback on-ramp metering control strategy that is proactive to the onset of congestion breakdown was developed. It uses a Kalman filter, a recursive forecasting algorithm, to predict traffic density and estimate on-ramp queue lengths. The adaptive control scheme works by controlling the traffic density on the mainline while considering on-ramp queues through weighting functions.

The design of the overall system was separated into three stages containing two major research components. In the first component, the design of dynamic linear models (DLM) and Kalman filters, used for prediction of the state of traffic, was performed. Next was the design of the metering control logic algorithm. Lastly, the two components were combined into a single integrated algorithmic program.

The remainder of the thesis is organized as follows. A literature review is presented in Chapter 2 of ramp metering, Kalman filtering, and stochastic capacity. Chapter 3 contains the methodological framework for state-space models, Kalman filtering, a review of traffic flow theory and feedback control theory, and the implemented ramp metering control design. Chapter 4 presents the modeling efforts for the DLM and Kalman filters in **R** (R Core Team, 2013), a programming language and environment for statistical computing. Chapter 5 describes the numerical programming of the control logic and the amalgamation of the prediction and control algorithms in MATLAB (MATLAB, 2013), a high-level language for numerical computation and programming. Chapter 6 is the concluding chapter which, contains a summary of the work performed and recommendations for future work.

CHAPTER 2: LITERATURE REVIEW

A literature review was conducted on existing ramp metering systems and the control algorithms they employ, Kalman filtering uses in transportation applications, and stochastic capacity. Besides that, properties that have to be taken into account when developing a proactive control strategy will be provided.

2.1 Ramp Metering

To combat the onset of breakdown, the most widely adopted freeway control is ramp metering. Ramp meters are traffic signals at the entrances of freeways that regulate the flow of vehicles onto the freeway. By regulating the amount and timing of vehicles, ramp meters break up platoons (i.e., groups) and reduce freeway demand. When a vehicle is given a green light, it is allowed to smoothly enter the flow of freeway traffic by theoretically taking advantage of existing gaps in the mainline traffic (Arnold, 1998).

The primary objective of ramp metering systems is to reduce freeway congestion; however, secondary objectives may be identified and accomplished with ramp metering such as the reduction of freeway crashes. The Federal Highway Administration's (FHWA) Ramp Management and Control Handbook advises practitioners consider the following seven aspects before determining a ramp metering plan (FHWA, 2006):

1. Metering strategy – A strategy should reflect the goals and objectives of the system but in general seeks to improve conditions on the freeway, while minimizing queuing and delay.

2. Geographic extent – The extent that metering occurs, i.e. at a single ramp that's isolated or over multiple ramps and will they be linked.
3. Metering approaches – Pre-timed or adaptive.
4. Metering Algorithms – Programming logic to determine metering rate.
5. Queue Management – How ramp queues will be maintained to an acceptable length and what is acceptable.
6. Flow Control – The manner and rate by which vehicles are allowed to enter a freeway ramp meter.
7. Signing – Appropriate signing needs to be implemented along the ramp as well as on nearby arterials to alert motorists to the presence and operation of ramp meters and to the specific driving instructions they need to perform when approaching a ramp.

2.1.1 Benefits and Impacts

There are more than 2,200 ramp meters in 29 metropolitan areas in the U.S. controlled by agencies with varying strategies and objectives for their use (FHWA, 2006). Because of this, benefits and impacts of metering is measured by the implementing agency or practitioners “metering philosophy” and may not be assessed similarly by the public.

Generally, the reported benefits of ramp meters include:

- Improved system operation.
 - Increased vehicle throughput.

- Increased vehicle speeds.
- Improved use of existing capacity.
- Improved safety.
 - Reduction in number of crashes and crash rate in merge zones.
 - Reduction in number of crashes and crash rate on the freeway upstream of the ramp/freeway merge zone.
- Reduced environmental effects.
 - Reduced vehicle emissions.
 - Reduced fuel consumption.
- Promotion of multi-modal operation.

The main disadvantage to ramp metering is that it can create long queues and delays on the on-ramps. Although metering can shift traffic to portions of the network where capacity is underutilized, the potential to inadvertently shift congestion to the arterial streets is possible. In a study aimed to measure the equity and efficiency of ramp meters, Levinson, Zhang, Das, and Sheikh, (2004) found the most efficient ramp control algorithm was also the least equitable one. The public's opposition to ramp metering has been linked to the equity issues associated with ramp metering policy (Yin, Liu, & Benouar, 2004).

Overall, there are two broad types of ramp metering control systems: local and coordinated. Local ramp metering consists of a section of freeway with one on-ramp in which the controller responds only to measurements in the vicinity of the ramp. Coordinated ramp metering consists of the application of metering at a series of on-ramps and may use available traffic measurements from greater portions of the freeway

(Smaragdis, Papageorgiou, & Kosmatopoulos, 2004). Within those systems, two methods of controlling ramp meters are pre-timed (also referred to as time-of-day or fixed-time) and traffic-responsive (FHWA, 2006).

Pre-timed metering is the simplest and cheapest form of ramp metering. Metering rates are determined from off-line, historical demands. In general, time-of-day's metering rate is independent of the existing traffic conditions. Traffic-responsive metering rates are determined or selected based on prevailing traffic conditions measured in real-time. Most modern ramp metering systems are traffic-responsive.

2.1.2 Local Ramp Metering

Local ramp metering considers an isolated section of the network that consists of a freeway segment with a single on-ramp and a controller that responds only to changes in the local conditions (Papageorgiou & Kotsialos, 2002; Kachroo & Ozbay, 2003; Smaragdis et al., 2004).

2.1.2.1 Demand-Capacity Strategy

Popular in North America, the local demand-capacity (DC) strategy calculates metering rates (Masher et al., 1975):

$$r(k) = \begin{cases} q_{cap} - q_{in}(k-1), & \text{if } o_{in}(k-1) \leq o_{cr} \\ r_{min}, & \text{else} \end{cases} \quad (2.1)$$

where $k=1,2,\dots$ is the discrete time index; $r(k)$ is the ramp flow (veh/hr) to be implemented during the new period k ; q_{cap} is the freeway capacity downstream of the ramp; $q_{in}(k-1)$ is the last measured upstream freeway flow measurement, upstream of the ramp; $o_{in}(k-1)$ is the last measured upstream freeway percent occupancy; o_{cr} is the

critical occupancy; and r_{\min} is a prespecified minimum ramp flow rate (Papageorgiou & Kotsialos, 2002). This strategy aims to add to the upstream flow $q_{in}(k-1)$ the amount of ramp flow $r(k)$ necessary to reach the downstream capacity q_{cap} (Smaragdis & Papageorgiou, 2003). If the occupancy out o_{out} becomes overcritical, the ramp flow is reduced to r_{\min} until traffic has dissolved. The r_{\min} is a parameter that would be determined by the agency controlling the system and is typically a function of storage capacity and vehicle arrival rate.

2.1.2.2 Occupancy Control

The occupancy control (OCC) strategy uses the same philosophy as the DC strategy, but uses occupancy based measurements for q_{in} . If the left-hand side of the fundamental diagram (Figure 1) is approximated by a straight line, q_{in} is:

$$q_{in} = \frac{v_f o_{out}}{g} \quad (2.2)$$

where v_f is the free speed of the freeway and g is the g-factor (Smaragdis & Papageorgiou, 2003).

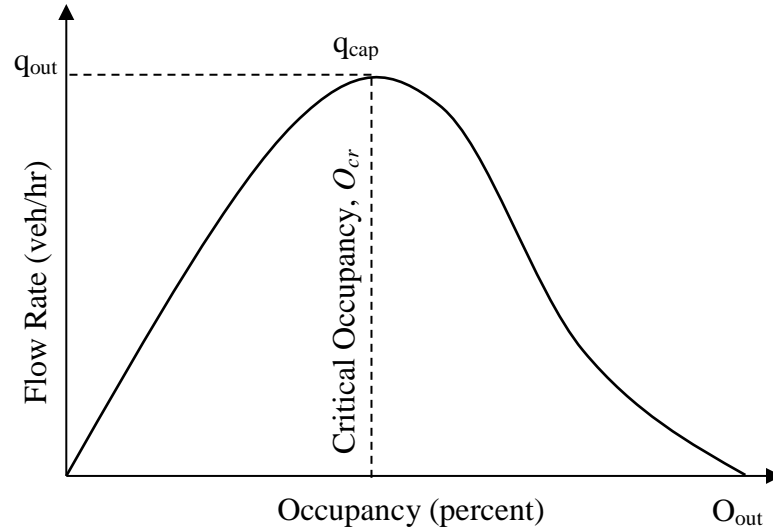


Figure 1. Fundamental Diagram with Left Side Approximated by a Straight Line
Substituting (2.2) into (2.1) gives:

$$r(k) = q_{cap} - \frac{v_f}{g} \times o_{out}(k-1) \quad (2.3)$$

Both DC and OCC strategies are considered feed forward, or open-loop. In open-loop systems, the system output is not used as input in the next iteration. This type of feed forward controller is blind to the control outcome. Conversely, in feedback control, or closed-loop control, the control input is based on the system output. In a closed-loop control system, output is fed back recursively so the control variable is a function of the output of a system (Kachroo & Ozbay, 2003).

2.1.2.3 ALINEA

The most widely used closed-loop local ramp metering algorithm is called Asservissement LINéaire d'Entrée Autoroutière (ALINEA) (Papageorgiou, Hadj-Salem, & Blosseville, 1991). ALINEA works by attempting to maximize throughput by

maintaining a desired occupancy, measured downstream of the ramp. ALINEA calculates metering rates by:

$$r(k) = r(k-1) + K_R [o_{des} - o_{out}(k)] \quad (2.4)$$

where $r(k)$ is the metering rate at time step k ; $K_R > 0$ is a regulator parameter (veh/hr); o_{des} is a set (desired) value for the downstream occupancy. ALINEA is a feedback control algorithm because it uses the previous time step ($k-1$) metering rate in the next iteration $r(k)$ and in control theory is known as an integral feedback regulator (Papageorgiou, Blosseville, & Haj-Salem, 1990b). The ramp metering rate needs to be converted into a cycle time with a green-phase and red-phase; typical to standard traffic signals (but without a yellow phase). For ALINEA, the green-phase duration is determined by:

$$g(k) = g(k-1) + K_R \frac{C}{r_{sat}} [o_{des} - o_{out}(k)], \quad g_{min} \leq g \leq g_{max} \quad (2.5)$$

where $g(k)$ is the green-phase duration at time interval k (seconds); $g(k-1)$ is the green-phase duration at previous time interval $k-1$ (seconds); C is the cycle duration (red-phase + green-phase) (seconds); r_{sat} is the ramp saturation flow (capacity flow) (veh/hr); g_{min} is the minimum green-phase duration (seconds); g_{max} is the maximum green-phase duration (seconds) (Papageorgiou, Hadj-Salem, & Middelham, 1997).

ALINEA requires a single detector station placed downstream of the merge area, has few parameters, and has not been very sensitive to the choice of the regulator parameter K_r (veh/hr) (Papageorgiou & Kotsialos, 2002). ALINEA has been one of the most widely used and effective algorithms and has required little modifications since its

inception in 1991, demonstrating superiority compared to the DC and OCC strategies (Smaragdis & Papageorgiou, 2003).

Other implemented coordinated ramp metering strategies include the Zone algorithm along I-35 East in Minneapolis/St. Paul, Minnesota (Stephanedes, 1994). In this metering strategy, a freeway network is divided into zones where the upstream end is typically represented by free flow conditions and the downstream end is at the location of a bottleneck (Zhang et al., 2001). Free flow speed (FFS) is the average speed a motorist would travel with no traffic that would affect their speed decisions.

A number of modifications and extensions of ALINEA have been presented (Smaragdis & Papageorgiou, 2003). FL-ALINEA, a flow based strategy, was introduced to provide alternatives to ALINEA's occupancy based measurements. In cases where uncertainties existed in the g -factor or where network-wide specification of target values was sought, Smaragdis and Papageorgiou (2003) claim it may be easier to specify target values for flows rather than occupancies. FL-ALINEA, an integral regulator, attempts to meter to the capacity q_{cap} ; however, because q_{cap} may underestimate the freeway capacity, FL-ALINEA is not recommended as a flow maximizing ramp metering strategy.

In some instances, sensors may exist only upstream of an on-ramp. UP-ALINEA is a feedback strategy based on estimates of downstream occupancy by means of upstream measurements. By combining FL-ALINEA and UP-ALINEA, a methodology using upstream measurements of flows is UF-ALINEA (Smaragdis & Papageorgiou, 2003).

The performance of the ALINEA algorithm is dependent upon the values selected for critical occupancy and the regulator gain. Sensitivity analysis performed in a

comparative study of control algorithms found ALINEA to be robust under critical occupancies ranging from 0.10 to 0.15 and regulator gains ranging from $K = 5,000$ to 30,000 (Chu, Liu, Recker, & Zhang, 2004). The above reported gain values are significantly more than the reported $K = 70$ in (Papageorgiou et al., 1991) and $K = 16$ (Papageorgiou et al., 1990b).

2.1.3 Coordinated Ramp Metering

Coordinated ramp metering (also referred to as system-wide) employs multiple ramp meters as part of a series of on-ramps where the response of all the ramps takes into account conditions beyond the local zone. Coordinated systems are designed to optimize flow through a corridor or network, rather than a single ramp. Coordinated algorithms can be divided into three types: cooperative, competitive, and integral (Zhang et al., 2001).

2.1.3.1 Cooperative Algorithms

Cooperative algorithms are similar to local except that changes in one metering rate affects metering rates of upstream ramps. An example of a cooperative algorithm is the Helper ramp algorithm used along the I-25 freeway in Denver, Colorado (Lipp, Corcoran, & Hickman, 1991).

2.1.3.2 Competitive Algorithms

In competitive ramp metering, two metering rates are computed; one based on local conditions and the other based on the network conditions, and the most restrictive one is chosen. Examples of the competitive algorithms include the Bottleneck algorithm on I-5 in Seattle, Washington (Jacobson, Henry, & Mehyar, 1989); FLOW (Jacobson et

al., 1989), and SWARM (Paesani, Kerr, Perovich, & Khosravi, 1997). SWARM consists of two independent competing algorithms. The first level of SWARM attempts to estimate the density trend based on past detector measurements by performing linear regression and a Kalman filtering process to forecast the traffic trend (Zhang et al., 2001). The second level of SWARM can actually be any traditional local algorithm traffic-responsive system.

2.1.3.3 Integral Algorithms

Integral ramp metering considers both local and system-wide traffic conditions to achieve some network objective, such as travel time through the network. The Sperry ramp metering algorithm implemented in 1985 in northern Virginia along I-395 and I-66 (Bogenberger & May, 1999) and the Fuzzy Logic algorithm in use today along I-405 Seattle, Washington (Meldrum & Taylor, 1995) are examples of integral algorithms. In the Fuzzy Logic algorithm, system-wide traffic data are converted into qualitative categories. So called ‘fuzzy rules’ are used to weight the qualitative categories and convert the “fuzzified” measurements into a metering level (Bogenberger & May, 1999). According to Zhang et al. (2001), this algorithm is theoretically attractive but not straightforward to implement and requires a great amount of effort to calibrate and tune, which when done improperly, performs very poorly.

Linear programming based algorithms (Yoshino, Sasaki, & Hasegawa, 1995) maximize/minimize an objective function based on a series of constraints. The METALINE algorithm is the ALINEA algorithm extended to multiple, coordinated ramps that make use of occupancy measurements $o_i(k)$, $i = 1, \dots, n$, to simultaneously calculate ramp metering rates $r_i(k)$, $i = 1, \dots, m$ (Papageorgiou et al., 1990b). The

metering rate of each ramp is computed through an analogous vector form of the ALINEA equation:

$$\mathbf{r}(k) = \mathbf{r}(k-1) - \mathbf{K}_1 [\mathbf{o}(k) - \mathbf{o}(k-1)] + \mathbf{K}_2 [\mathbf{O}_{des} - \mathbf{O}(k)] \quad (2.6)$$

where $\mathbf{r}(k)$ is the vector of metering rates for m controlled ramps at time step k ; $\mathbf{o}(k)$ is the vector of n available measured occupancies; $\mathbf{O}(k)$ is a subset of \mathbf{o} for which the desired occupancy \mathbf{O}_{des} is given at m controlled ramps, respectively. $\mathbf{K}_1, \mathbf{K}_2$ are two gain matrices. In control theory, the control law (2.5) is called a proportional-integral feedback regulator (Papageorgiou et al., 1990b) and will be shown why in Section 3.3.2.

Gokasar et al. (2013) proposed two coordinated strategies, D-MIXCROS and C-MIXCROS, that explicitly consider queues through weighting factors, $w_{1(i)}$ and $w_{2(i)}$, applied to on-ramps and the mainline sections where they enter, for $i = 1, 2, \dots, n$ sections. The proportional-derivative feedback logic attempts to minimize the error:

$$e(t) = w_{1(1)} \left| \rho_1(t) - \rho_{cr(1)} \right| + w_{2(1)} queue_{ramp1}(t) + w_{1(2)} \left| \rho_2(t) - \rho_{cr(2)} \right| + w_{2(2)} queue_{ramp2}(t) + \dots + w_{1(n)} \left| \rho_n(t) - \rho_{cr(n)} \right| + w_{2(n)} queue_{rampn}(t) \quad (2.7)$$

where ρ and ρ_{cr} are the measured density and critical density (veh/mi) of the freeway sections, respectively; $queue_{ramp1,2}$ are calculated queue lengths. The parameters are determined so that they ensure maximization of freeway throughput ($w_{1(i)}$) without creating long queues on the on-ramps ($w_{2(i)}$) (Gokasar et al., 2013).

2.1.4 Queue Management

Possibly the greatest unappealing aspects of ramp metering are ramp delays, ramp queues, and queue spillback into arterials. Most modern ramp metering algorithms incorporate some form of queue management. Queue detectors are placed near the beginning of the ramp, or at critical locations where excess queues would cause undesirable effects.

A strategy most ramp meter algorithms take when a queue is detected is to adjust the metering rate to a less aggressive level, as to disperse the queue more quickly. Another solution known as queue flushing, or queue override, completely disables ramp metering and resumes when queues return to acceptable levels. This has the potential to cause oscillatory patterns. For example, when a metering rate becomes more restrictive, an excessive ramp queue can form and queue flushing is activated, temporarily disabling the meter. Those vehicles inundate the freeway, exacerbating congestion, causing a more restrictive metering rate and repeating the cycle.

2.2 Kalman Filtering

Kalman filters have been used in transportation engineering to estimate traffic variables (Fei, Lu, & Liu, 2011; Gazis & Liu, 2003; Ghosh, 1978; Ojeda, Kibangou, & Wit, 2013; Okutani & Stephanedes, 1984; Portugais & Khanal, 2014) as well as extended Kalman filters (Tampere & Immers, 2007; van Hinsbergen, Schreiter, Zuurbier, van Lint, & van Zuylen, 2012; Wang & Papageorgiou, 2005; Wang et al., 2009; Wang, Papageorgiou, & Messmer, 2008). The Kalman filter is a set of mathematical equations used to recursively estimate the state of a dynamic process in a way that minimizes the mean of the squared error (Welch & Bishop, 2006).

The term “filter” is actually a bit of a misnomer and actually implies data processing algorithm. The algorithm estimates the state of a discrete time linear dynamic system containing noise with a two-step feedback control process. In the first step, a prediction is made of the current value state variable along with their uncertainties.

When a measurement of the state process is made, the new information is incorporated to obtain an improved prediction. In this sense, the equations for the Kalman filter can be thought of as a time-update (prediction) and measurement-update (correction) feedback control.

2.3 Stochastic Capacity

Capacity is a random quantity that is difficult for practitioners to determine and apply in real-world settings. Volume is described by the demand, the actual number of observed or predicted vehicles, and restricted by the capacity. Capacity can be defined as theoretical/design capacity or operational capacity (Wu, Michalopoulos, & Liu, 2010). Freeway operational capacity has been defined by Minderhoud, Botma, and Bovy (1997) as the capacity value representing the actual maximum flow rate of the roadway. Haboian (1993) described a reduction in capacity due to congestion resulting from a reduction in traffic flow. When demand exceeds capacity, there will be some reduction in the capacity of the freeway segments, normally 3 percent to 10 percent, and as high as up to 24 percent (Aghdashi, 2013). Although some studies have reported that bottlenecks can support very high flows prior to their activation, these high flows have typically been observed only for time periods that are short relative to the rush (Cassidy, 2003).

The capacity of a freeway is traditionally treated as a constant value in traffic engineering guidelines around the world, such as the most widely used, The Highway

Capacity Manual (HCM) (Brilon, Geistefeldt, & Regler, 2005; Transportation Research Board, 2010). However, many studies have shown that different capacities can be observed on freeways even under constant external conditions (Brilon et al. 2005; Geistefeldt and Brilon 2009; Wu et al., 2010; Elefteriadou et al. 2011).

Research investigating the stochastic nature of freeway capacity and breakdowns have been applied to ramp metering algorithms (Stratified Zone Metering, COMPASS) by modifying them to incorporate the breakdown probability in the control scheme (Elefteriadou et al., 2011; Geistefeldt & Brilon, 2009). Modifications have included adjusting the metering rates based on the maximum capacity prior to the onset of breakdown. These studies have shown that the freeway breakdown phenomenon is stochastic, meaning it is a probabilistic event and can occur over a range of flow values (Jia, 2013; Elefteriadou et al., 2011) and capacity values (Wu et al., 2010). The term ‘breakdown’ describes the transition from free-flowing traffic to congestion. A more common intermediary state, termed ‘slow-and-go waves,’ is used to describe less than congested traffic and characterized by speeds of 20-40 mph and somewhat erratic acceleration and braking.

Brilon et al. (2005) developed a nonparametric model using the “Product Limit Method” (PLM), which is based on the statistics of lifetime data analysis (Kaplan and Meier, 1958), to derive a theoretical transformation between capacities identified for different interval durations. The method, however, could not estimate an appropriate distribution function so various distribution types were examined. By comparing different types of functions based on the value of maximum likelihood, it was determined that the Weibull distribution was the best fit. Brilon et al. (2005) assert that higher

demand volumes are less likely to be observed in the field since there is a higher probability that a breakdown occurred during a preceding interval but claim traffic breakdowns in succeeding intervals are independent of one another.

Elefteriadou et al. (2011) identified critical bottlenecks as those where congestion was recurring due to merging operations and was distinguished by low speeds propagating upstream, whereas free-flowing (or near free-flowing) conditions occurred downstream. Using a nonparametric technique, two ramp metering algorithms (Stratified Zone Metering; Minnesota and COMPASS Ontario, Canada) were modified to incorporate the breakdown probability as the basic foundation for control activities. The distribution function of breakdown volume was generated using the PLM (Kaplan and Meier, 1958; Brilon et al., 2005; Geistefeldt and Brilon, 2009). Again, the complete distribution function was obtained using a Maximum-Likelihood technique and was fit to a log-normal distribution. At the highest observed flows, the probability of breakdown using this method did not reach 0.25, a calculation that makes the results questionable.

CHAPTER 3: METHODOLOGY

The proposed solution to ramp metering uses a prediction algorithm, known as a Kalman filter, and a control scheme that regulates timing of the vehicles released onto the freeway. The algorithmic scheme then can be thought of as a recursive prediction algorithm producing forecasts that the control algorithm uses as inputs. The Kalman filter is used to predict the state process (state of traffic) through observations collected by roadway sensors. The Kalman filter is employed within a special class of time-series models called state-space models.

Understanding traffic flow theory is critical for a successful ramp metering strategy. A review of traffic flow theory is given in Section 3.3.

3.1 State-Space Framework

The state-space framework considers a time-series as the output of a dynamic system perturbed by random disturbances and ones in which parameters are allowed to vary over time (Casdagli, 1992). State-space models can be used for modeling univariate non-stationary time-series that allow for natural interpretation as a result of trend and seasonal (periodic) components (Durbin & Koopman, 2012; Petris, Petrone, & Campagnoli, 2009). The state-space local level model is a time-series where observations can be modeled as random fluctuations around a stochastic level described by a random walk. A random walk is defined as a process where the current value of a variable is composed of the past value plus an error term defined as a white noise

sequence (Shumway & Stoffer, 2011). A special case of general state-space models that are linear and Gaussian are also called dynamic linear models.

3.1.1 Dynamic Linear Model

The main tasks for the DLM were to make inferences on the unobserved traffic state and predict future observations based on part of the observation sequence. The DLM models were constructed and estimated using maximum likelihood estimation (MLE) in **R**, the Statistical Environment and Language (R Core Team, 2013). Kalman (1960) presented a new look into stochastic processes and forecasting using the “state transition” method of analysis of dynamic systems known as the Kalman filter. In dynamic state-space models, the Kalman filter provides the formulas for updating our current inference on the state vector \mathbf{x}_k as new data \mathbf{y}_k become available. The use of the DLM in this research allowed for trends and seasonal patterns to be captured and the matrices structure determined to pass to the Kalman filter. The details of those matrices will be discussed in Chapter 4.

3.2 Kalman Filtering

The Kalman filter works by making a prediction of the future and comparing the estimate with real-time measurements. Along with the prediction, an error covariance is calculated. A Kalman filter is an optimal recursive data processing algorithm, meaning that predictions are based on only the previous time-step’s prediction and the filter does not require all previous data to be stored and reprocessed with new measurements. The filter is optimal in the sense that it minimizes the variance of the estimation error at each iteration process. When the next measurement is taken, the algorithm calculates a correction of the state prediction using the new measurement along with the error

covariance. The recursive algorithm uses only the current measurement and error covariance allowing for low computational cost and on-line forecasting.

The general problem that the Kalman filter addresses is the estimation of the state \mathbf{x}_k of a discrete-time controlled process that is governed by the general state equation

$$\mathbf{x}_k = \mathbf{G}_k \mathbf{x}_{k-1} + \mathbf{w}_k \quad (3.1)$$

based on measurements \mathbf{y}_k , according to the observation equation:

$$\mathbf{y}_k = \mathbf{F}_k \mathbf{x}_k + \mathbf{v}_k \quad (3.2)$$

where \mathbf{G}_k and \mathbf{F}_k are known matrices and \mathbf{v}_k and \mathbf{w}_k are Gaussian independent white noise sequences, where

$$\mathbf{w}_k \sim \mathcal{N}(0, \mathbf{Q}_k) \quad (3.3)$$

and

$$\mathbf{v}_k \sim \mathcal{N}(0, \mathbf{R}_k) \quad (3.4)$$

The Kalman filter can be thought of as a recursive two-stage prediction and measurement update algorithm with the prediction stage equations given by:

State estimate (*a priori*):

$$\hat{\mathbf{x}}_{k|k-1} = \mathbf{G}_k \hat{\mathbf{x}}_{k-1|k-1} \quad (3.5)$$

Error covariance estimate (*a priori*):

$$\mathbf{P}_{k|k-1} = \mathbf{G}_k \mathbf{P}_{k-1|k-1} \mathbf{G}_k^T + \mathbf{Q}_k \quad (3.6)$$

The predicted state estimate is also known as the *a priori* state estimate because it does not include information from the current time step. In the measurement update

stage, the prediction is combined with the current observation information to refine the state estimate and is known as the *a posteriori* state estimate. The innovation is defined as the difference between the observation (measurement) and its prediction using the information available when the new measurement is taken. It is known as an “innovation” because it effectively provides new information. The measurement update equations are given by:

Measurement innovation (or residual):

$$\tilde{\mathbf{y}}_k = \mathbf{y}_k - \mathbf{F}_k \hat{\mathbf{x}}_{k|k-1} \quad (3.7)$$

The innovation covariance:

$$\mathbf{S}_k = \mathbf{F}_k \mathbf{P}_{k|k-1} \mathbf{F}_k^T + \mathbf{R}_k \quad (3.8)$$

The weight of the correction term is given by the Kalman filter gain:

$$\mathbf{K}_k = \mathbf{P}_{k|k-1} \mathbf{F}_k^T \mathbf{S}_k^{-1} \quad (3.9)$$

which is the adaptive coefficient and can be regarded as an information tuner (Fei et al., 2011; Petris et al., 2009).

State estimate (*a posteriori*):

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k \tilde{\mathbf{y}}_k \quad (3.10)$$

Error covariance estimate (*a posteriori*):

$$\mathbf{P}_{k|k} = \mathbf{P}_{k|k-1} - \mathbf{K}_k \mathbf{F}_k \mathbf{P}_{k|k-1} \quad (3.11)$$

Commonly it is assumed that the noise terms \mathbf{w}_k and \mathbf{v}_k are Gaussian and under these conditions the KF will provide the optimal estimate of \mathbf{x}_k . Otherwise, the KF is the best linear estimate, which is often sufficient in stochastic linear estimation.

3.3 Traffic Flow Theory

Traffic flow is characterized by three fundamental characteristics: speed v , flow q , and density ρ and are mathematically related:

$$q = v \times \rho \quad (3.12)$$

Flow is defined as the rate at which vehicles travel through a particular point or freeway segment, typically expressed in vehicles per hour (veh/hr). Density is expressed in units of vehicles per unit of distance, typically vehicles per mile (veh/mi). Density is a difficult variable to measure; however, it is a very useful performance measure (Elefteriadou, 2014). Because density is difficult to measure, occupancy is often a surrogate for density and is measured:

$$D = \frac{5,280 \times O}{L_v + L_d} \quad (3.13)$$

where O , occupancy, is defined as the percentage of time that the detection zone of an instrument, often an inductance loop, is occupied by a vehicle; L_v is the average length of a vehicle (feet), L_d is the length of the detector (feet). Inductance loop traffic detectors are the most common freeway traffic detector and work by transmitting an electrical current through wires installed in the pavement. When a vehicle passes over the loops, it causes a change in the wire's inductance and sends a pulse to a controller signifying the

presence of a vehicle. The fundamental diagrams (FD) of traffic flow graphically gives the relation between two of the fundamental variables of traffic flow.

The de facto guide for procedural guidelines and concepts for computing capacity and quality of service of various highway facilities is the 2010 Highway Capacity Manual. The 2010 HCM uses a three-regime speed-flow model where flows can be in the undersaturated, oversaturated, or queue discharge regions as shown in Figure 2.

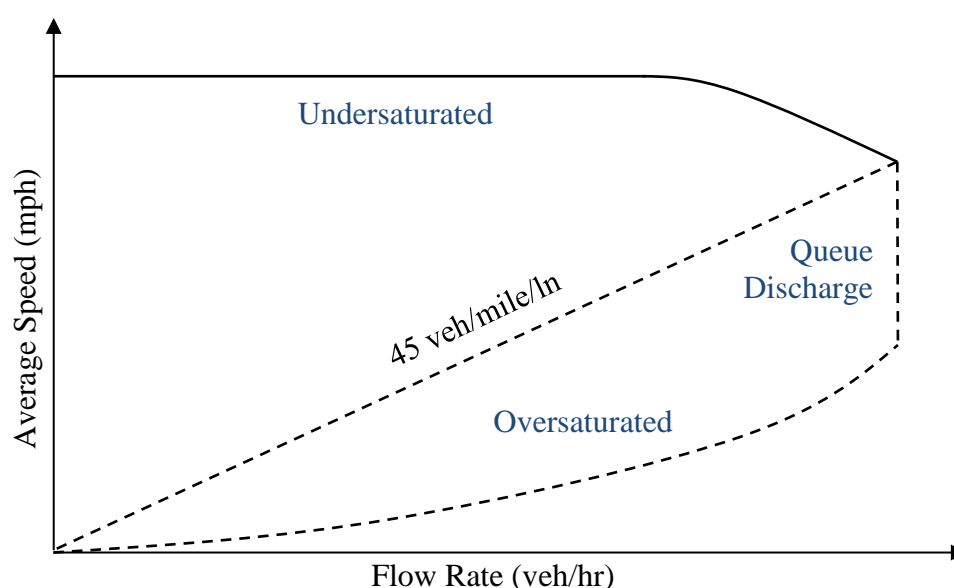


Figure 2. HCM Three-Regime Speed-Flow Relationship

Undersaturated conditions are typically represented by free flow conditions (or FFS) whereas oversaturated conditions are the result from a bottleneck. In oversaturated conditions the arrival flow rate is greater than the capacity at a point along the road segment. Queue discharge is flow from a bottleneck that accelerates back to FFS in the absence of a downstream bottleneck (Edara, 2010). The HCM diagram above shows that speeds are constant for low flows and begins to decrease as flow reaches 1,350 to 1,750 passenger cars/hr/lane (Transportation Research Board, 2010). This curve was developed

with the assumption that capacity is reached when density is 45 veh/mi (Elefteriadou, 2014).

The relationship between flow and density in the 2010 HCM model is parabolic in the uncongested region and linear in the congested flow region, as depicted in Figure 3. Jam density, represented by the intersection of the curve and x-axis, is the density of vehicles that are stopped in traffic due to severely congested conditions. The critical density (or optimal density) corresponds to the maximum flow.

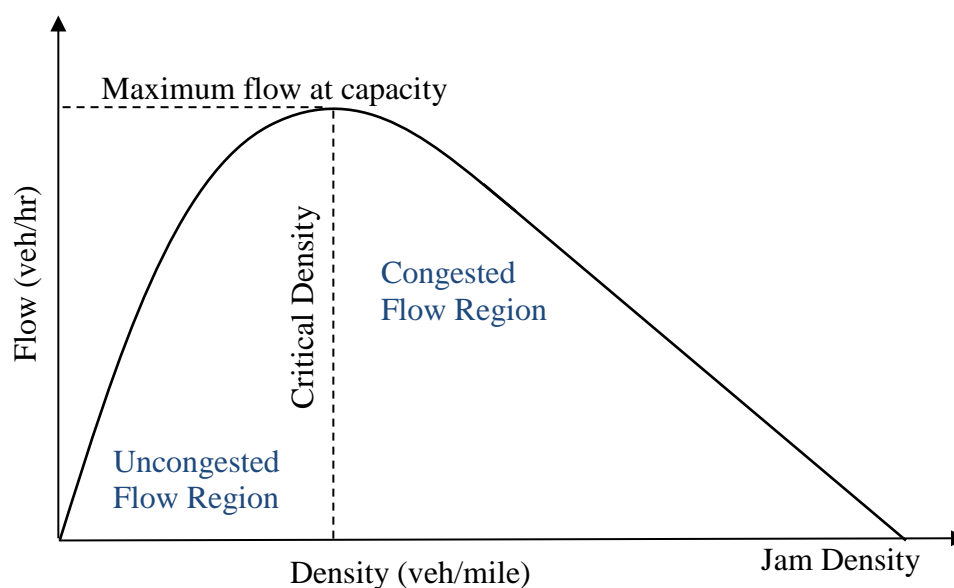


Figure 3. HCM Flow-Density Relationship

In the undersaturated region, one of the model parameters is FFS, which is estimated by averaging all speed observations with flow rates less than 1000 (pc/h/ln) (Sajjadi, 2013; Transportation Research Board, 2010). All HCM calculations and analysis are based on the assumption that the capacity of different freeway segments is a deterministic value. A flaw in this approach is that it does not exclude congested observations and considers them in FFS estimation.

3.3.1 Traffic Bottlenecks and Shockwave Theory

As stated, traffic bottlenecks are locations where congestion originates and can be stationary or moving. Stationary bottlenecks occur due to road design, accidents, or traffic poorly timed traffic signals. Moving bottlenecks are caused by slow moving vehicles that cause disruption in traffic. The propagation of a congestion wave-front from a bottleneck is known as a shockwave. Shockwaves can be created when platoons of vehicles attempt to enter the freeway causing turbulence in the mainline stream of vehicles.

Lighthill and Whitham (1955) introduced shockwave theory, which describes the propagation of different traffic states and provides an analytical method to describe the fundamental relationship between speed, flow, and density. In shockwave theory, regions between two traffic states characterized by different speeds, densities, and/or flow rates boundary is propagated at speed ω .

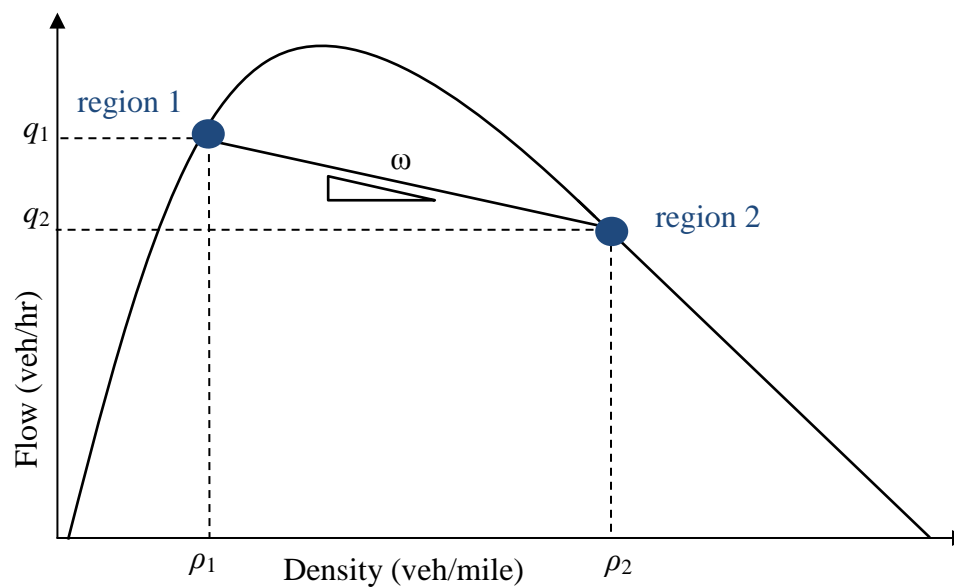


Figure 4. Graphical Representation of Shockwave Speed

If one considers a line that connects two traffic regions 1 and 2 in the flow-density diagram, then slope of the line represents the speed of the shockwave as determined by:

$$\omega = \frac{q_2 - q_1}{k_2 - k_1} \quad (3.14)$$

where q_2, q_1 and k_1, k_2 are the flows and densities between regions 1 and 2, respectively as shown in Figure 4.

3.4 Feedback Control Theory

Control theory is the study of behavior of dynamical systems with a view towards controlling them (Simrock, 2007). In closed-loop (or feedback) systems, the variable being controlled is a function of the output of the system. Therefore our control variable, the metered ramp outflow, should be a function of the state of the system, traffic density and queue length (Kachroo & Ozbay, 2003).

In describing feedback controls, three components generally make up the elementary feedback control system as shown in Figure 5: a plant (the system or process to be controlled), a sensor to measure the output of the plant, and a controller to generate input to the plant (Doyle, Francis, & Tannenbaum, 1990).

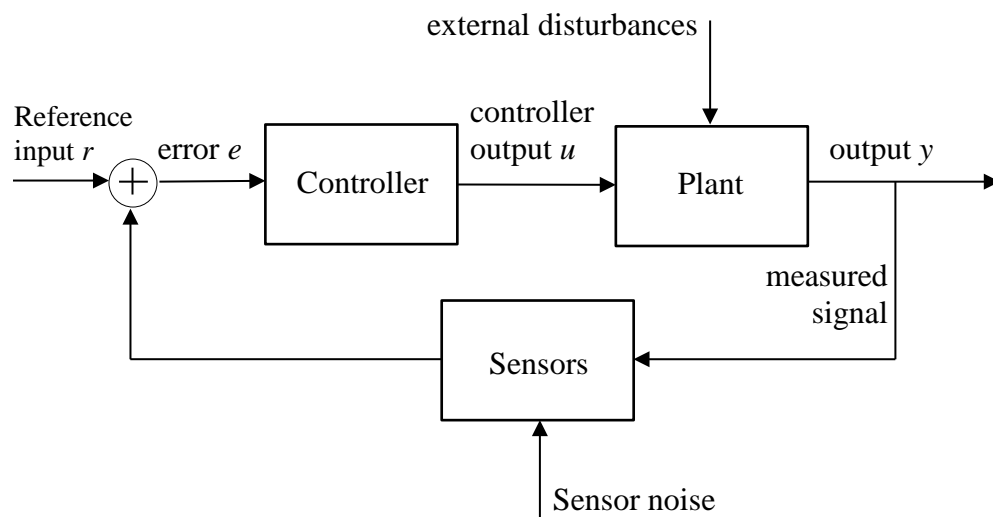


Figure 5. Components of the Elementary Feedback Control System

The model of the system to be controlled, the freeway traffic system, is represented by the plant block. The three outside signals (reference input, external disturbances, and sensor noise) are exogenous inputs. The output from the plant should approximate some prespecified function of the reference input, and it should do so in the

presence of disturbance, sensor noise, and uncertainty in the plant (Doyle et al., 1990). To achieve this purpose, the manipulated plant input is changed at the directive of the controller. One such controller is the widely used feedback Proportional-integral-derivative controllers (PID) or some variant of them.

The proportional-integral-derivative terms are actions that the controller takes in response to the feedback error and its basic form is (Astrom & Murray, 2012):

$$u(t) = k_p e(t) + k_i \int_0^t e(\tau) d\tau + k_d \frac{de}{dt} \quad (3.15)$$

where u is the control signal and e is the control error, the difference between the desired input r and the actual output y . Along with the PID controller response, tuning parameters known as gains are used to adjust the response. An explanation of each of the above term follows.

3.4.1 Proportional Response

The proportional control term produces an output value that is proportional to the error:

$$P_{out} = k_p e(t) \quad (3.16)$$

where k_p , a tuning parameter, is the proportional gain constant used to adjust the P-control response P_{out} . The P-term can be thought of as adjusting for the “present” error (Araki, 2002). When the P-gain k_p is high, the action taken by the controller results in a large change in u for a given change in e . Conversely, the same is true for a low k_p ; the controllers response to a given error will be small.

3.4.2 Integral Response

The integral response is proportional to the magnitude and duration of the error and can be thought of as the accumulation of the “past” error that should have been previously corrected (Araki, 2002). It is calculated by the integral of the error:

$$I_{out} = k_i \int_0^t e(\tau) d\tau \quad (3.17)$$

where k_i , a tuning parameter, is the integral gain constant used to adjust the I-control response I_{out} . When the I-term is added to the P-term, it accelerates the movement of the control action towards the desired input r . However, because the I-term is the response to accumulated past errors, it can cause the present value to exceed the desired input r (Bisen & Sharma, 2012).

The control error for ALINEA is $e(k) = o_{des} - o_{out}(k)$; rewriting (2.4) gives:

$$r(k) - r(k-1) = K_R e(k) \quad (3.18)$$

Dividing both sides by the sampling time Δt and taking the limits gives:

$$\lim_{\Delta t \rightarrow 0} \frac{r(k) - r(k-1)}{\Delta t} = \lim_{\Delta t \rightarrow 0} \frac{K_R e(k)}{\Delta t} \quad (3.19)$$

Which gives:

$$\dot{r}(t) = K_R e(k) \quad (3.20)$$

The ALINEA control law is obtained by integration of (3.20) (Kachroo & Ozbay, 2003), thus making it an integral-response.

3.4.3 Derivative Response

The derivative-response has the effect of making changes to the control output that are dependent on the rate of change of the process error (i.e., its first derivative with respect to time) and is:

$$D_{out} = k_d \frac{de}{dt} \quad (3.21)$$

where k_d , a tuning parameter, is the derivative gain constant used to adjust the D-control response D_{out} . The derivative-term can be thought of as a “prediction” of future error based on the current slope of error (Araki, 2002). The Proportional-integral-derivative control structure is shown below in Figure 6.

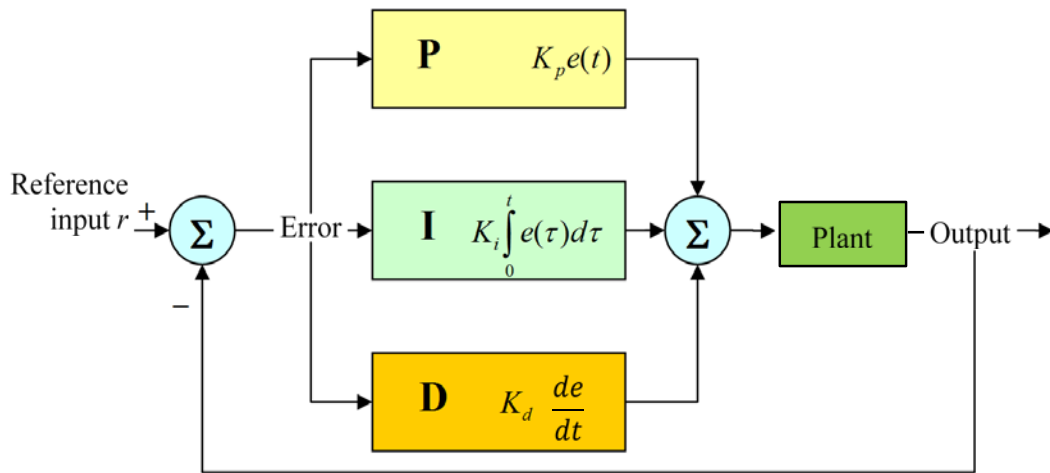


Figure 6. Proportional-Integral-Derivative Control Structure

3.5 Feedback Coordinated Ramp Metering Control Design

The aim of our control law is to maximize the throughput of the freeway sections while taking into consideration on-ramp queues. Considerations for design of the feedback controller are not only maximizing throughput and balancing equity for ramp

delays, but also coordinating between the two ramps. The design uses two coordinated feedback-based control strategies, D-MIXCROS and C-MIXCROS, presented by Gokasar et al. (2013).

In these feedback strategies, the coordinated ramp metering problem is expressed as a problem of controlling the traffic density on the mainline while minimizing ramp queues through weights w_i . The weighting parameters w_i determine how much influence the mainline freeway density and queue lengths on the ramps should be given. A model for the freeway traffic flow is presented next.

The freeway is discretized into 2 sections each of which contains one on-ramp as shown in Figure 7 with traffic detectors shown as dashed lines.

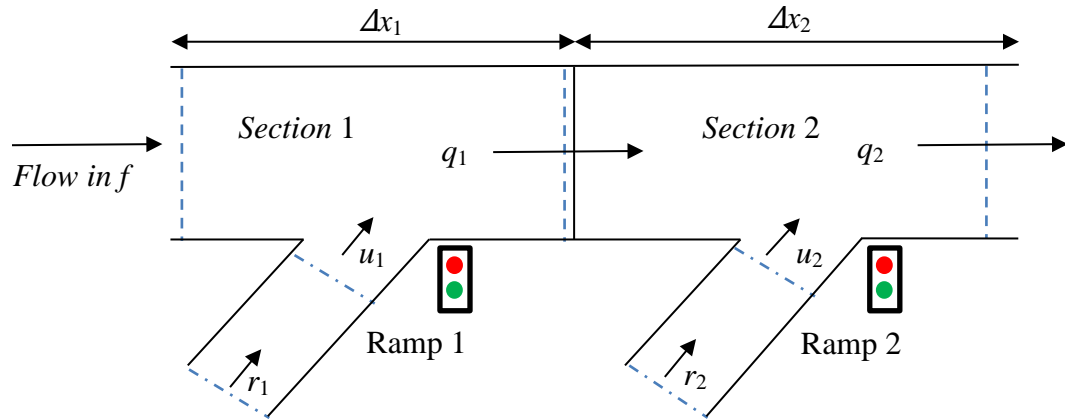


Figure 7. Discretized Freeway with Meter and Traffic Sensors Located

A section can be defined as a portion of the freeway between two mainline detectors and contains one on-ramp. As seen in the above figure, the flow out of the first section q_1 is the flow in for the next section. Flow in f (veh/hr) represents the flow into section 1 of the mainline. The ramp demand is represented by r_1 and r_2 and the controlled metered

outflow to the mainline is u_1 and u_2 . The length of sections 1 is Δx_1 ; length of section 2 is Δx_2 ; length of ramp 1 is Δx_{r1} ; length of ramp 2 is Δx_{r2} , respectively. In discrete time step k , the ramp 1 and ramp 2 queue lengths l_1 and l_2 , and the traffic density ρ_i of each section changes through the equations:

$$\begin{aligned}
 \rho_1(k+1) &= \rho_1(k) + \frac{T}{\Delta x_1} [-q_1(k) + u_1(k) + f(k)] \\
 l_1(k+1) &= l_1(k) + \frac{T}{\Delta x_{r1}} (r_1(k) - u_1(k)) \\
 \rho_2(k+1) &= \rho_2(k) + \frac{T}{\Delta x_2} [-q_2(k) + u_2(k) + q_1(k)] \\
 l_2(k+1) &= l_2(k) + \frac{T}{\Delta x_{r2}} (r_2(k) - u_2(k))
 \end{aligned} \tag{3.22}$$

where T is the sampling time interval, q_{i+1} is the downstream flow, and q_i is the upstream flow. The first and third equations describe the conservation of vehicles, which holds strictly in any case. The second and fourth equations compute the queue growth. A queue is formed when the ramp demand r exceeds the metered output u . The control design is presented in the proceeding sections.

3.5.1 Control Objective

The objective of the algorithm is to keep vehicle density levels near the critical density so as to allow the maximum flow throughput possible while constraining the development of on-ramp queues. With this, ramp metering rates are designed to become stricter as the mainline approaches critical density, the threshold between congested and uncongested conditions. The objective of the feedback control design is to make the error term go to zero asymptotically:

$$\lim_{k \rightarrow \infty} e(k) = 0 \quad (3.23)$$

Equation 3.23 can be satisfied by use of a proportional-derivative state feedback control logic that follows the closed-loop dynamics:

$$e(k+1) + Ke(k) = 0 \quad (3.24)$$

where K is the control gain, and e is the error represented by the error function:

$$e(k) = w_1 |\rho_1(k) - \rho_{cr1}| + w_2 l_1(k) + w_3 |\rho_2(k) - \rho_{cr2}| + w_4 l_2(k) \quad (3.25)$$

where $l_{1,2}$ are queue lengths on ramps 1 and 2; w_1 and w_2 are the weighting factors assigned to section 1 mainline and ramp 1, respectively with w_1 and $w_2 = 1$; w_3 and w_4 are the weight factors assigned to section 2 mainline and ramp 2, respectively with w_3 and $w_4 = 1$; ρ_1 is the density of section 1 and ρ_2 is the density of section 2; ρ_{cr1} is the critical density of section 1 and ρ_{cr2} is the critical density of section 2.

If a nonzero weight is given to either section weights w_1 or w_3 or the ramp queue weights w_2 or w_4 , the controller tries to reduce on-ramp queues while attempting to keep mainline density near critical. If zero is given to w_2 or w_4 , then the controller will not take into account ramp queues. This way, the buildup of queues can be handled more meticulously and in a proactive manner than a queue flushing or queue override system.

The system can be in four regions. In region 1, the traffic density of both sections is greater than the critical density. The error function in this region is:

$$e(k) = w_1 (\rho_1(k) - \rho_{cr1}) + w_2 l_1(k) + w_3 (\rho_2(k) - \rho_{cr2}) + w_4 l_2(k) \quad (3.26)$$

Incrementing (3.26) gives:

$$e(k+1) = w_1(\rho_1(k+1) - \rho_{cr1}) + w_2 l_1(k+1) + w_3(\rho_2(k+1) - \rho_{cr2}) + w_4 l_2(k+1) \quad (3.27)$$

Using (3.22) here gives:

$$\begin{aligned} e(k+1) = & w_1 \left[\rho_1(k) - \rho_{cr1} + \frac{T}{\Delta x_1} (-q_1(k) + u_1(k) + f(k)) \right] \\ & + w_2 \left[l_1(k) + \frac{T}{\Delta x_{r1}} (r_1(k) - u_1(k)) \right] \\ & + w_3 \left[\rho_2(k) - \rho_{cr2} + \frac{T}{\Delta x_2} (q_2(k) + u_2(k) + q_1(k)) \right] \\ & + w_4 \left[l_2(k) + \frac{T}{\Delta x_{r2}} (r_2(k) - u_2(k)) \right] \end{aligned} \quad (3.28)$$

Rearranging the right-hand side of (3.28) and dropping time index k for brevity gives:

$$\begin{aligned} e(k+1) = & w_1 \left[\rho_1 - \rho_{cr1} + \frac{T}{\Delta x_1} (-q_1 + f) \right] \\ & + w_2 \left[l_1 + \frac{T}{\Delta x_{r1}} r_1 \right] + \left[w_1 \frac{T}{\Delta x_1} - w_2 \frac{T}{\Delta x_{r1}} \right] u_1 \\ & + w_3 \left[\rho_2 - \rho_{cr2} + \frac{T}{\Delta x_2} (-q_2 + q_1) \right] \\ & + w_4 \left[l_2 + \frac{T}{\Delta x_{r2}} r_2 \right] + \left[w_3 \frac{T}{\Delta x_2} - w_4 \frac{T}{\Delta x_{r2}} \right] u_2 \end{aligned} \quad (3.29)$$

Equation 3.29 is a difference equation and can be written as:

$$e(k+1) = F(k) + u(k) \quad (3.30)$$

where

$$\begin{aligned}
F(k) = & w_1 \left[\rho_1 - \rho_{cr1} + \frac{T}{\Delta x_1} (-q_1 + f) \right] \\
& + w_2 \left[l_1 + \frac{T}{\Delta x_{r1}} r_1 \right] \\
& + w_3 \left[\rho_2 - \rho_{cr2} + \frac{T}{\Delta x_2} (-q_2 + q_1) \right] \\
& + w_4 \left[l_2 + \frac{T}{\Delta x_{r2}} r_2 \right]
\end{aligned} \tag{3.31}$$

and

$$u(k) = \left[w_1 \frac{T}{\Delta x_1} - w_2 \frac{T}{\Delta x_{r1}} \right] u_1 + \left[w_3 \frac{T}{\Delta x_2} - w_4 \frac{T}{\Delta x_{r2}} \right] u_2 \tag{3.32}$$

In region 2, the traffic density in both sections is less than or equal to the critical density. In region 3, the traffic density of the first section is greater than the critical density and less in the second section. Region 4 has traffic density less than the critical density in the first section and greater in the second section (Gokasar et al., 2013; Kachroo & Ozbay, 2003). The derivation of control function in the other three regions is not shown but can be found in Kachroo and Ozbay (2003).

3.5.1.1 Overall Control Law

To satisfy the closed-loop dynamics (3.24), the overall control law for the coordinated ramp control is:

$$u(k) = -F(k) - Ke(k) \tag{3.33}$$

where:

$$\begin{aligned}
F(k) = & w_1 \operatorname{sgn} \left(\rho_1 - \rho_{cr1} + \frac{T}{\Delta x_1} (-q_1 + f) \right) \\
& + w_2 \left[l_1 + \frac{T}{\Delta x_{r1}} r_1 \right] \\
& + w_3 \operatorname{sgn} \left(\rho_2 - \rho_{cr2} + \frac{T}{\Delta x_2} (-q_2 + q_1) \right) \\
& + w_4 \left[l_2 + \frac{T}{\Delta x_{r2}} r_2 \right]
\end{aligned} \tag{3.34}$$

where “sgn” denotes the signum function and

$$\begin{aligned}
u(k) = & \left[\operatorname{sgn}(\rho_1 - \rho_{cr1}) w_1 \frac{T}{\Delta x_1} - w_2 \frac{T}{\Delta x_{r1}} \right] u_1 \\
& + \left[\operatorname{sgn}(\rho_2 - \rho_{cr2}) w_3 \frac{T}{\Delta x_2} - w_4 \frac{T}{\Delta x_{r2}} \right] u_2
\end{aligned} \tag{3.35}$$

Substituting (3.33) into (3.30) satisfies the desired dynamics of (3.24). The control output Equation 3.35 does not give the control laws, but it provides the condition that the control variables u_1, u_2 should satisfy.

The feedback, traffic-responsive ramp metering control system can be seen in Figure 8. The reference input signal is the desired traffic density ρ_{cr} , which is the objective density for the system.

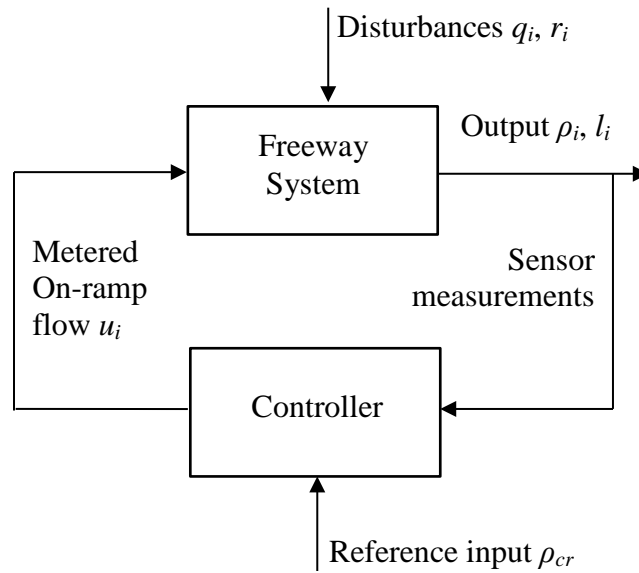


Figure 8. Feedback Control System

In our traffic system, the freeway flows f and q_i and ramp demand r_i are disturbances, measurable in real-time with detectors, such as inductive loops. The control law is designed as decoupled (D-MIXCROS) or coupled (C-MIXCROS) (Gokasar et al., 2013; Kachroo & Ozbay, 2003).

3.5.2 Decoupled D-MIXCROS

In the decoupled setting, the ramp controls are treated independently as isolated ramp meter control systems. The F term in (3.34) is “decoupled” so that $F = F_1 + F_2$

With the time index k added, these become:

$$\begin{aligned}
F_1(k) &= w_1 \operatorname{sgn} \left(\rho_1(k) - \rho_{cr1} + \frac{T}{\Delta x_1} (-q_1(k) + f) \right) \\
&\quad + w_2 \left[l_1(k) + \frac{T}{\Delta x_{r1}} r_1(k) \right] \\
F_2(k) &= w_3 \operatorname{sgn} \left(\rho_2(k) - \rho_{cr2} + \frac{T}{\Delta x_2} (-q_2(k) + q_1) \right) \\
&\quad + w_4 \left[l_2(k) + \frac{T}{\Delta x_{r2}} r_2(k) \right]
\end{aligned} \tag{3.36}$$

Decoupling (3.35), the decoupled control law becomes:

$$\begin{aligned}
u_1(k) &= \left[\operatorname{sgn}(\rho_1(k) - \rho_{cr1}) w_1 \frac{T}{\Delta x_1} - w_2 \frac{T}{\Delta x_{r1}} \right]^{-1} (-F_1(k) - Ke_1(k)) \\
u_2(k) &= \left[\operatorname{sgn}(\rho_2(k) - \rho_{cr2}) w_3 \frac{T}{\Delta x_2} - w_4 \frac{T}{\Delta x_{r2}} \right]^{-1} (-F_2(k) - Ke_2(k))
\end{aligned} \tag{3.37}$$

where the error terms are defined:

$$\begin{aligned}
e_1(k) &= w_1 |\rho_1(k) - \rho_{cr1}| + w_2 l_1(k) \\
e_2(k) &= w_3 |\rho_2(k) - \rho_{cr2}| + w_4 l_2(k)
\end{aligned} \tag{3.38}$$

The application of (3.36) results in the decoupled closed-loop dynamics:

$$\begin{aligned}
e_1(k+1) + Ke_1(k) &= 0 \\
e_2(k+1) + Ke_2(k) &= 0
\end{aligned} \tag{3.39}$$

3.5.3 Coupled C-MIXCROS

In the coupled setting, the ramp and mainline are coordinated through weight factors w_i and the controllers are coupled between one another through distribution factors. In this scheme, the control effort is divided between the two controllers by distribution factors α_1 and α_2 . The distribution factors α_1 and α_2 can be thought of as providing the communication between the on-ramp systems (Gokasar et al., 2013). For example, if congestion was propagating from the Eagle Road EB on-ramp (controller 2) towards the EB loop on-ramp (controller 1), the control effort could be increased at controller 1 making it more restrictive. The coupled control law is:

$$\begin{aligned} u_1(k) &= \left(\text{sgn}(\rho_1(k) - \rho_{cr1}) w_1 \frac{T}{\Delta x_1} - w_2 \frac{T}{\Delta x_{r1}} \right)^{-1} \alpha_1 (-F(k) - Ke(k)) \\ u_2(k) &= \left(\text{sgn}(\rho_2(k) - \rho_{cr2}) w_3 \frac{T}{\Delta x_2} - w_4 \frac{T}{\Delta x_{r2}} \right)^{-1} \alpha_2 (-F(k) - Ke(k)) \end{aligned} \quad (3.40)$$

where $\alpha_1 + \alpha_2 = 1$, the error term is defined by (3.25) and

$$\begin{aligned} F(k) &= w_1 \text{sgn} \left(\rho_1(k) - \rho_{cr1} + \frac{T}{\Delta x_1} (-q_1(k) + f) \right) \\ &\quad + w_2 \left[l_1(k) + \frac{T}{\Delta x_{r1}} r_1(k) \right] \\ &\quad + w_3 \text{sgn} \left(\rho_2(k) - \rho_{cr2} + \frac{T}{\Delta x_2} (-q_2(k) + q_1) \right) \\ &\quad + w_4 \left[l_2(k) + \frac{T}{\Delta x_{r2}} r_2(k) \right] \end{aligned} \quad (3.41)$$

3.6 Field Site and Data Collection

3.6.1 Site Selection

Interchanges from Gowen Road to Franklin Road (Canyon County) were examined as potential locations for ramp metering. Average daily traffic volumes on ramp interchanges and mainline throughput were examined as well as travel times for the corresponding segments. Volume and travel time data were obtained from the Idaho Transportation Department (ITD) and COMPASS, respectively. Detailed analysis of travel time with respect to changes in volume was not performed. The initial screening of interchanges was those with significant ramp volume with respect to mainline/arterial volume. Ramps with no practicable solution due to right-of-way, limited access, and freeway-to-freeway connection were eliminated.

The connector originating from the downtown area was not selected for these reasons. Gowen Road, Vista Avenue, and Orchard Street interchanges do not have traffic volumes that warrant ramp metering. The Cole-Overland ramp interchange experiences average daily traffic of approximately 16,000 vehicles (ITD, 2012) that may either merge onto I-84 or I-184. Additional data is needed to justify metering at this location; however, it is believed to be unnecessary due to acceleration lanes of approximately 2,625 feet and 5,165 feet for the entrances to I-84 and I-184, respectively.

A bottleneck condition exists at the area between the Eagle Road EB-loop and EB on-ramp. Eagle Road and Meridian Road have the highest arterial traffic volumes in the corridor, occurring north of the interchanges. At the Eagle Road interchange, the presence of traffic signals 1,300 feet apart at St. Luke's Road and the northern on/off ramps can have the potential to cause significant platooning for vehicles entering on the

drifted westward faster than the employment center increasing the number of vehicle miles travelled for commutes. For these reasons, the Eagle Road interchange's EB on-ramps were selected for ramp metering.

3.6.2 Data Collection and Description

Data for traffic volume, occupancy, vehicle classification, and average lane speed were not available on I-84 near Meridian Road and Eagle Road interchanges and needed to be collected at resolutions previously unavailable.

Six radar based (SmartSensor105™ from Wavetronix) sensing devices were chosen to capture five-minute averaged volume, speed, and occupancy data from November 6 through November 15 in 2013 on the EB direction of I-84 from the Meridian Road interchange to the Eagle Road interchange as shown in Figure 10.

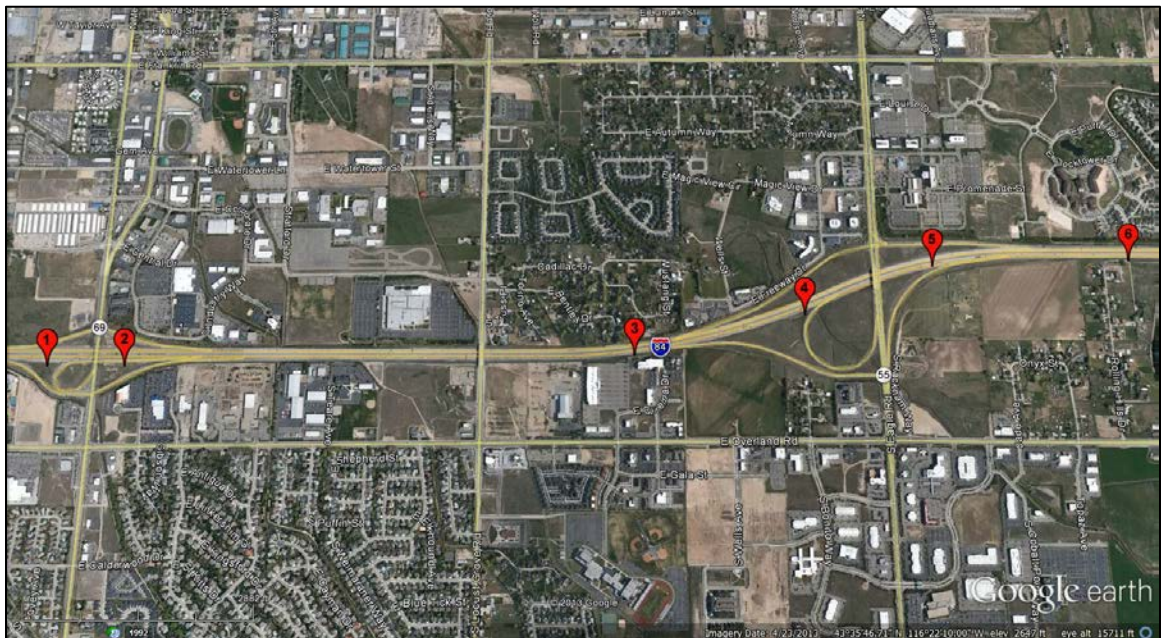


Figure 10. Approximate EB locations of Wavetronix Radar Detectors

Data collection for the WB direction took place from November 20 through November 29, 2013. The locations for WB sensors are below in Figure 11. Sensors were deployed based on knowledge of existing prevailing traffic conditions to capture the effects of downstream bottlenecks, exit and entrance ramps, and weaving and merging due to ramps. Shockwave perturbations between bottlenecks and slow-and-go wave oscillations were observed in the study area from Meridian Road to Eagle Road, most notably.

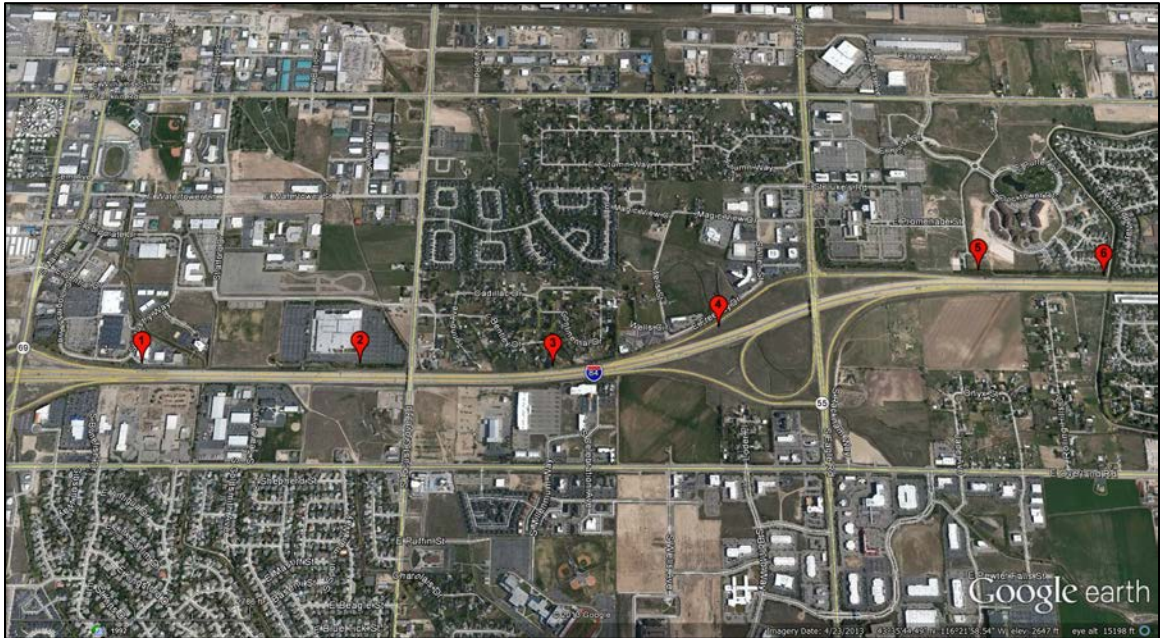


Figure 11. Approximate WB locations of Wavetronix Radar Detectors

Displayed in Figure 12 are 24-hour time-series plots for traffic speeds, flows, and densities collected at EB site 6. At 8:00 AM, the peak of the rush-hour, it is characterized by high traffic densities and flows accompanied by significant speed reductions. In order to minimize the turbulence caused from the decreased speeds, the density needs to be controlled.

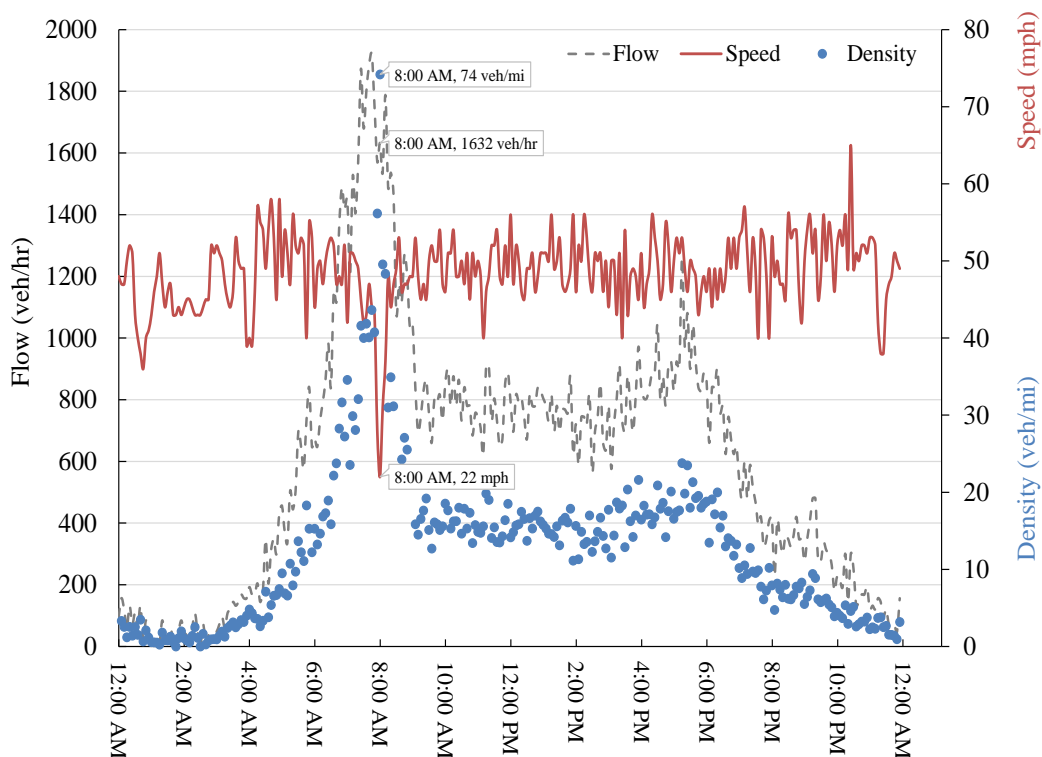


Figure 12. Time-Series Plots for Traffic Speeds, Flows, and Density, EB Site 6

In the above example, if the demand were 1632 veh/hr and the speeds were desired to be maintained at 52 mph, the traffic density would need to be approximately 31.4 veh/mi. By controlling the density, speeds and maximum flows can be maintained.

Data collected from Bluetooth sensors were also utilized. Permanent installations of traffic monitoring equipment based on Bluetooth technology are in place throughout sections of I-84. The Bluetooth devices collect and time-stamp media access control (MAC) addresses from Bluetooth devices in vehicles traveling on a road and, by matching these addresses collected at the two end points, can yield travel times and speeds between those points.

CHAPTER 4: DLM & KALMAN FILTER SIMULATION

R was used to model and forecast traffic speeds and volumes collected from radar based sensors. **R** was used mainly for parameter estimation of variance of noise sequences \mathbf{w}_t and \mathbf{v}_t , state matrix \mathbf{G}_t , and observation matrix \mathbf{F}_t . Kalman filtering was performed initially in **R** on the DLMs, then completed in MATLAB, where the ramp metering controls were constructed. The data set for this task contains five-minute average lane speeds collected from midnight November 11 through noon on November 14, 2013.

4.1 R Language and Environment

One of the difficulties in formulating the Kalman filter can be determining the state and observation matrices and their error covariances. In fact, in most engineering literature, it is common to assume that the model structure is known, except for disturbances of the noise and state process. In time-series analysis, the physical process of the underlying states of the dynamic system is often less apparent and what is most relevant is the problem of forecasting (Petris et al., 2009). In this setting, model building tends to be more difficult and even when a state-space model is obtained, there can be parameters or unknown values remaining in the model.

R was used to solve this problem. **R** is a free, open-source software programming language and environment, mainly used for statistical computing. Many user-created packages exist that allow for specialized modeling, with the focus here being on time-

series analysis, specifically state-space models. DLMs were produced to model traffic volumes and traffic speeds that were recorded during the radar data collection effort and originated from the EB Site 4 Lane 1 (See Figure 10) location.

4.2 R Model Specification, Parameter Estimation: Traffic Volume

Dynamic linear models allow for periodic and seasonal trend components to be captured efficiently. The **dlm** package provides *build* functions for creating standard types of DLMs including: **dlmModARMA** for ARMA process; **dlmModPoly** for n th order polynomial DLMs; **dlmModReg** for linear regression of a dlm; **dlmModSeas** for periodic and seasonal factors; **dlmModTrig** for representing periodicity with Fourier-form (Petris, 2009, 2010). These functions can be combined to create DLMs with those various components that particular function represent. See (Petris et al., 2009; Petris, 2009) for a complete description.

One of the simplest DLMs is the already mentioned random walk (RW) plus noise model. This type of model was constructed in **R** to model and forecast freeway traffic volumes as follows (**R** code given in Lucida Console font size 10):

```
buildvol <- function(theta) {
  dlmModPoly(order = 1, dV = theta[1],
  dW = theta[2])}
(4.1)
```

The above model represents a random walk plus noise model where dV (analogous to (3.4)) represent the unknown variance of the observation noise and dW (analogous to (3.3)) represents the unknown variance of the system noise. By using the “function(theta)” argument, one is essentially saying any “theta” that follows is an unknown parameter to be estimated with MLE.

Three types of models were constructed in **R**: a RW with noise, a RW with noise and seasonal components, and a RW with Fourier-form seasonal components. The model specification was completed by using *build* functions that served as skeleton models with empty parameters to be later estimated by MLE. Once the model parameters were fit with the MLE of their parameters, the KF algorithm was applied. The remainder of this section describes the MLE and results for the three models and then applies the KF algorithm to the fitted model and presents the results.

4.2.1 Maximum Likelihood Estimation

The model specifications and parameter estimation for the DLMs were completed in **R** using the package **d1m** (Petris, 2010). This package focuses on Bayesian analysis of DLMs. The package also includes functions for determining the parameters of a DLM with MLE and for Kalman filtering. Estimating unknown parameters in a DLM requires numerical techniques that rely on **optim**, an optimizer that is built-in with **R**.

The function **d1mMLE** uses the MLE to estimate the unknown parameters starting at user defined initial values. In general, **d1mMLE** evaluates a user defined function *build*, such as (4.1), and uses it to define the negative loglikelihood function (Petris, 2010). **Optim** then minimizes the negative loglikelihood function. In **R**, this is coded:

$$\begin{aligned} \text{fitvol} <- \text{d1mMLE}(\text{vol}, \text{parm} = \text{c}(0, 150), \text{buildvol}, \\ \text{hessian} = \text{T}, \text{lower} = \text{rep}(1\text{e-}4)) \end{aligned} \quad (4.2)$$

where **vol** is the data set; **parm** gives the vector of initial values; **buildvol** is the *build* function (4.1); **hessian=T** forces **optim** to return a numerically evaluated Hessian at the

minimum and **lower** (Petris, 2009, 2010) recommends using $1e-4$ defines the lower bound for the parameter space.

It's recommended to repeat the optimization process multiple times, starting at different parameter values and to verify the Hessian of the negative loglikelihood at the maximum is positive definite (Petris, 2009, 2010). He also recommends checking the inverse of the Hessian matrix of the negative loglikelihood function at the MLE. It provides an estimate of the asymptotic variance of the MLE where the standard errors can be derived from the diagonal elements (Petris, 2009, 2010). In **R**, this is coded:

```
[1] modvol <- buildvol(fitvol$par) ## Fitted model
[2] fitvol$convergence ## Check Convergence
[3] drop(W(modvol)) ## System Variance
[4] drop(V(modvol)) ## Observation Variance
[5] hs <- hessian(function(x) dlMLL(vol, buildvol(x)), fitvol$pa)
[6] all(eigen(hs, only.values = TRUE)$values > 0) ## Positive Definite?
[7] aVar <- solve(hs) ## Asymptotic Variance / Covariance Matrix
[8] sqrt(diag(aVar)) ## Standard Errors
```

where comments follow **##**. Line [1] represents the fitted model with the MLE of the parameters; [2] checks convergence of the fitted model; [3]-[4] display the system and observation variances; [5] – [8] checks the asymptotic variance of the MLE and standard errors. The results of [1] – [8] are below where the symbol > designates a line of code entered into **R** and the response of that command (if any) is followed by *[ans]*:

```

> modvol <- buildvol(fitvol$par) ## Fitted model
> fitvol$convergence ## Check Convergence
[ans] (TRUE)
> drop(W(modvol)) ## System Variance
[ans] 11.84381
> drop(V(modvol)) ## Observation Variance
[ans] 61.15916
> hs <- hessian(function(x) dlmLL(vol, buildvol(x)), fitvol$pa)
> all(eigen(hs, only.values = TRUE)$values > 0) ## Positive Definite?
[ans] TRUE
> aVar <- solve(hs) ## Asymptotic Variance / Covariance Matrix
> sqrt(diag(aVar)) ## Standard Errors
[ans] 2.662735 1.234964

```

4.2.2 Seasonal Models

Models that show a cyclical behavior or “seasonality” can be modeled by their seasonal factors or a Fourier-form representation of the seasonality. An example of what a seasonal time-series model may look like in transportation engineering is the periodic behavior of rush hour traffic. In fact, recurrent congestion conditions could be considered to be daily components of a weekly seasonal pattern. A seasonal model can be thought of having s periods. In the above example, if the observation period is one-hour, the recurrent congestions may occur every 12 periods, or every 12 hours. Meaning similar congestion conditions may appear at 7:00 AM and 7:00 PM every day. In **R**, this is coded into the *build* function:

```

buildvolseas <- function(theta) {
  dl mModPoly(order = 1, dV = theta[1],
  dW = theta[2]) +
  dl mModSeas(12)}

```

(4.3)

Finding the MLE of unknown parameters in (4.3) in **R**:

```

fitvolseas <- dl mMLE(vol, parm = c(0, 150),
  buildvolseas, hessian = T, lower = rep(1e-4))

```

(4.4)

Fitting the model with the MLE of parameters, verifying convergence, viewing the system and observation variance, checking the Hessian positive definite, and standard errors gives:

```

> modvolseas <- buildvolseas(fitvolseas$par) ## Fitted model
> fitvolseas$convergence ## Check Convergence
[ans] 0(TRUE)
> drop(W(modvolseas)) ## System Variance
[ans] 11.66658
> drop(V(modvolseas)) ## Observation Variance
[ans] 57.30299
> hs <- hessian(function(x) dl mLL(vol, buildvolseas(x)), fitvolseas$par)
> all(eigen(hs, only.values = TRUE)$values > 0) ## Positive Definite?
[ans] TRUE
> aVar <- solve(hs) ## Asymptotic Variance / Covariance Matrix
> sqrt(diag(aVar)) ## Standard Errors
[ans] 2.648271 1.235673

```

The Fourier-form representation usually allows for a more realistic representation of real-world seasonal phenomena; although, more modeling parameters are needed. A seasonal time-series might be represented as $Y_t = g(t-1) + v_t$, where $g(\cdot)$ is a periodic

function and v_t is the observation noise (Petris et al., 2009). Consider a zero-mean periodic function $g(t)$ having even period s , which can be written as:

$$g(t) = \sum_{j=1}^{s/2} \underbrace{a_j \cos(t\omega_j) + b_j \sin(t\omega_j)}_{j\text{th harmonic}} \quad (4.5)$$

where $\omega_j = 2\pi j/s$ are the Fourier frequencies. In **R**, this is coded into the *build* function:

$$\begin{aligned} \text{buildvolTri g} <- \text{function}(\text{theta}) \{ \\ & \text{dlmModPoly}(\text{order} = 1, \text{dV} = \text{theta}[1], \\ & \text{dW} = \text{theta}[2]) + \\ & \text{dlmModTri g}(s = 12, q = 6) \} \end{aligned} \quad (4.6)$$

where s is the number of periods and q is the number of harmonics. Finding the MLE of unknown parameters in (4.6) in **R**:

$$\begin{aligned} \text{fitvolTri g} <- \text{dlmMLE}(\text{vol}, \text{parm} = \text{c}(0, 150), \\ & \text{buildvolTri g}, \text{hessian} = \text{T}, \text{lower} = \text{rep}(1e-4)) \end{aligned} \quad (4.7)$$

Fitting the model with the MLE of parameters, verifying convergence, viewing the system and observation variance, checking the Hessian positive definite, and standard errors gives:


```

> modvolTri g <- buildvolTri g(fitvolTri g$par) ## Fitted model
> fitvolTri g$convergence ## Check Convergence
[ans] 0(TRUE)
> drop(W(modvolTri g)) ## System Variance
[ans] 11.66042
> drop(V(modvolTri g)) ## Observation Variance
[ans] 60.21902
> hs <- hessian(function(x) dlMLL(vol, buildvolTri g(x)), fitvolTri g$pa)
> all(eigen(hs, only.values = TRUE)$values > 0) ## Positive Definite?
[ans] TRUE
> aVar <- solve(hs) ## Asymptotic Variance / Covariance Matrix
> sqrt(diag(aVar)) ## Standard Errors
[ans] 2.610215 1.202532

```

4.2.3 KF Results: Traffic Volume RW Model

The fitted model **modvol** is:

$$\begin{aligned}
 \mathbf{G} &= [1] \\
 \mathbf{W} &= [11.844] \\
 \mathbf{F} &= [1] \\
 \mathbf{V} &= [61.159]
 \end{aligned}
 \tag{4.8}$$

where **G** is the system matrix and **W** is its noise variance; **F** is the observation matrix and **V** is its noise variance. The Kalman filter recursive algorithm was applied to (4.8) and the results of the one-step-ahead forecasts are shown in Figure 13.

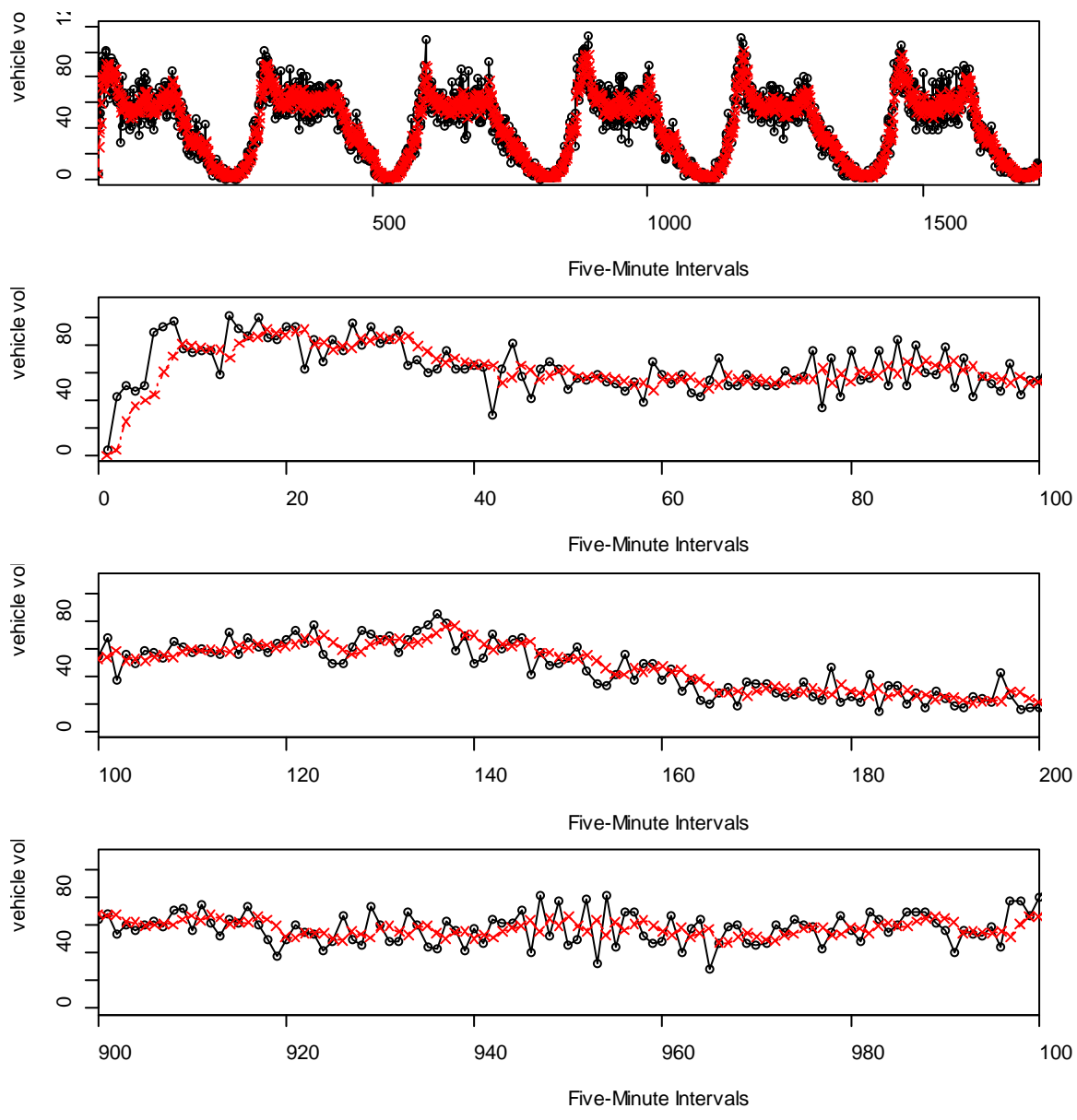


Figure 13. Kalman Filter Applied to Traffic Volume Random Walk with Noise Model

The top plot of Figure 13 is the entire filtering process of observed data and one-step ahead forecasts. The plot clearly shows 6 days of observations where one day is 288 five-minute time steps. As shown in the second plot, the filter converged to the approximate observed state at time index (TI) 15. The MLE of the state noise variance was able to track the observed state relatively well; however, when large changes in

measurements occur rapidly, the filter's response is limited. In general, this limitation is controlled by the ratio of the measurement noise to state noise, known as the signal-to-noise ratio.

After a prediction is made *a priori*, a measurement is taken and compared with the estimate and an innovation, or residual, is generated. Although the innovation is an error, it is aptly termed an innovation because of the fact that the new data reveals information that the filter uses to improve its estimates. The amount of corrective action taken in response to this innovation is weighted by the Kalman gain. Consequently, the Kalman gain depends upon the covariances of errors between the predicted and actual state and measurement.

4.2.4 KF Results: Traffic Volume RW with Seasonal Component Model

The fitted model **modvalseas** is:

$$\begin{aligned}
 \mathbf{G} &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \\
 \mathbf{W} &= \begin{bmatrix} 11.903 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
 \mathbf{F} &= [1 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] \\
 \mathbf{V} &= [55.747]
 \end{aligned} \tag{4.9}$$

The Kalman filter recursive algorithm was applied to (4.9) and the results of the one-step-ahead forecasts are shown in Figure 14. A seasonal component was added to the RW plus noise model to capture the effects of recurring traffic. As seen in the top plot of Figure 14, the traffic clearly exhibits patterns that repeat daily. A seasonal component was added to the RW model to exploit this regularity; however, the seasonal component did not improve the model.

The seasonal component is specified by describing the frequency at which this regularity is occurring. Obviously, with five-minute data, a frequency of 288 periods

would suggest that the traffic was regularly occurring every 24 hours. The computing resources were not available to model this period since the size of the state \mathbf{G} and noise \mathbf{W} matrices would have been 288×288 . A 12-hour period was selected to attempt to capture the seasonal component as shown in Figure 14.

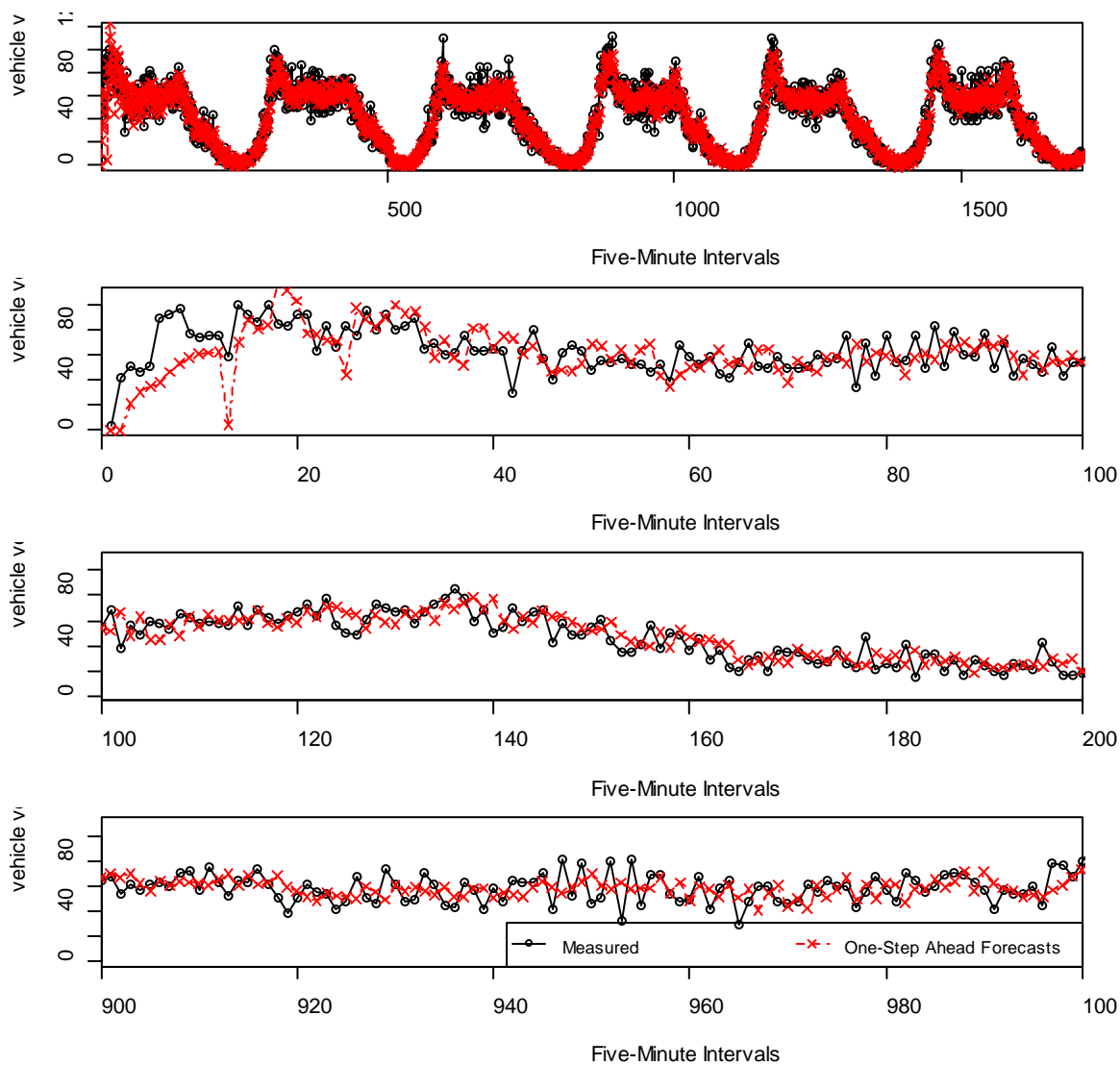


Figure 14. Kalman Filter Applied to Traffic Volume Random Walk with 12-Period Seasonal Component Model

4.2.5 KF Results: Traffic Volume RW with Fourier-Form Model

The fitted model **modvolTrig** is:

$$\begin{aligned}
 \mathbf{G} &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.87 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.5 & 0.87 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0.87 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.87 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2.8e-16 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 2.8e-16 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.5 & 0.87 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.87 & -0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.87 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.5 & -0.87 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \\
 \mathbf{W} &= \begin{bmatrix} 11.760 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
 \mathbf{F} &= [1 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1] \\
 \mathbf{V} &= [59.395]
 \end{aligned} \tag{4.10}$$

The Kalman filter recursive algorithm was applied to (4.10) and the results of the one-step-ahead forecasts are shown in Figure 15.

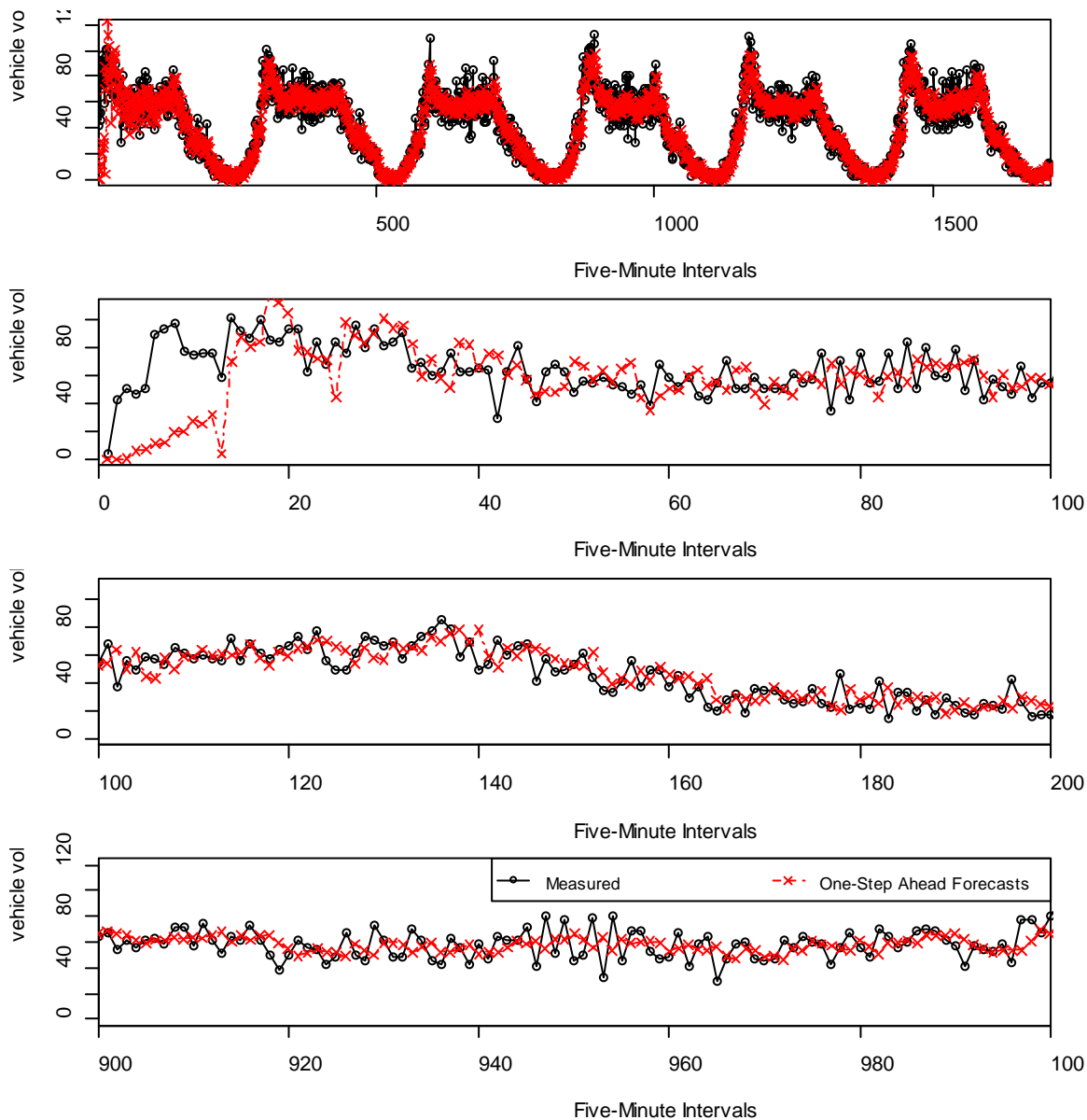


Figure 15. Kalman Filter Applied to Traffic Volume Random Walk with Fourier-form Seasonal Component Model

Since any periodic function on the real numbers can be approximated by a sum of harmonic functions, known as Fourier sums, seasonality can be modeled by a combination of sine waves (Petris et al., 2009). This seasonal representation is known as the Fourier-form and usually allows for a better representation of the seasonality in real-world phenomena (Petris et al., 2009). The model was not an improvement over the RW

plus noise model; however, the results for this representation are given for demonstrative purposes only.

4.3 R Model Specification, Parameter Estimation: Traffic Speeds

The methodology presented in the previous section to model traffic volumes was applied to the traffic speeds data and not shown to avoid redundancy. A random walk plus noise model was used as the base model, then seasonal components were added as seasonal factors and as Fourier-form. The data used for the traffic speed modeling effort originated from the EB Site 4 Lane 1 (See Figure 10) location. The data, containing 720 observations, was five-minute average lane speeds collected from midnight November 12 through noon on November 14, 2013.

A random walk was constructed to model and forecast freeway traffic speeds using the *build* function as follows:

$$\begin{aligned} \text{buildspd1} <- \text{function}(\text{theta}) \{ \\ & \text{dlmModPoly}(\text{order} = 1, \text{dV} = \text{theta}[1], \\ & \text{dW} = \text{theta}[2]) \} \end{aligned} \quad (4.11)$$

4.3.1 Maximum Likelihood Estimation

The parameters of the model (4.11) were estimated with MLE as follows:

$$\begin{aligned} \text{fitspd1} <- \text{dlmMLE}(\text{spd1}, \text{parm} = \text{c}(.2, 25), \text{buildspd1}, \\ \text{hessian} = \text{T}, \text{lower} = \text{rep}(1\text{e-}4)) \end{aligned} \quad (4.12)$$

Fitting the model with the MLE of parameters, verifying convergence, viewing the system and observation variance, checking the Hessian positive definite, and standard errors gives:


```

> modspd1 <- buildspd1(fitspd1$par) ## Fitted model
> fitspd1$convergence ## Check Convergence
[ans] 0(TRUE)
> drop(W(modspd1)) ## System Variance
[ans] 9.130866
> drop(V(modspd1)) ## Observation Variance
[ans] 4.367125
> hs <- hessian(function(x) dlMLL(spd1, buildspd1(x)), fitspd1$par)
> all(eigen(hs, only.values = TRUE)$values > 0) ## Positive Definite?
[ans] TRUE
> aVar <- solve(hs) ## Asymptotic Variance / Covariance Matrix
> sqrt(diag(aVar)) ## Standard Errors
[ans] 0.7491558 1.1587165

```

4.3.2 Seasonal Models

Traffic speeds are less likely to exhibit trends that can be modeled than traffic volumes. A similar volume of traffic may travel a road on any given day from week to week, but not at the same speed. This is why traffic speeds are a good indication of the quality of flow that is occurring in the traffic stream. The data was fit to one-hour frequency periods, rather than five-minute, to attempt to capture some seasonality that is not possible in 5-minute observations. A seasonal model was fit to traffic speeds in \mathbf{R} :

$$\begin{aligned}
 \text{buildspdseas} <- \text{function}(\text{theta}) \{ \\
 & \text{dlmModPoly}(\text{order} = 1, \text{dV} = \text{theta}[1], \\
 & \text{dW} = \text{theta}[2]) + \\
 & \text{dlmModSeas}(12)\}
 \end{aligned} \tag{4.13}$$

Finding the MLE of unknown parameters in (4.13) in \mathbf{R} :

$$\begin{aligned} \text{fitspdseas} &<- \text{dlmMLE}(\text{spd2}, \text{parm} = \text{c}(.1, 25), \\ &\text{buildspdseas}, \text{hessian} = \text{T}, \text{lower} = \text{rep}(1\text{e-}4)) \end{aligned} \quad (4.14)$$

Fitting the model with the MLE of parameters, verifying convergence, viewing the system and observation variance, checking the Hessian positive definite, and standard errors gives:

```
> modspdseas <- buildspdseas(fitspdseas$par) ## Fitted model
> fitspdseas$convergence ## Check Convergence
[ans] 0(TRUE)
> drop(W(modspdseas)) ## System Variance
[ans] 10.65023
> drop(V(modspdseas)) ## Observation Variance
[ans] 1.475796
> hs <- hessian(function(x) dlmLL(spd2, buildspdseas(x)), fitspdseas$par)
> all(eigen(hs, only.values = TRUE)$values > 0) ## Positive Definite?
[ans] TRUE
> aVar <- solve(hs) ## Asymptotic Variance / Covariance Matrix
> sqrt(diag(aVar)) ## Standard Errors
[ans] 0.8018066 1.3085438
```

The Fourier-form representation of seasonal components was coded into the *build* function in **R** as follows:

$$\begin{aligned} \text{buildspdTrig} &<- \text{function}(\text{theta}) \{ \\ &\quad \text{dlmModPoly}(\text{order} = 1, \text{dV} = \text{theta}[1], \\ &\quad \text{dW} = \text{theta}[2]) + \\ &\quad \text{dlmModTrig}(s = 12, q = 6) \} \end{aligned} \quad (4.15)$$

where s is the number of periods and q is the number of harmonics. Finding the MLE of unknown parameters in (4.15) in **R**:

$$\begin{aligned} \text{fitspdTrig} &<- \text{dlmMLE}(\text{spd1}, \text{parm} = \text{c}(.1, 25), \\ &\text{buildspdTrig}, \text{hessian} = \text{T}, \text{lower} = \text{rep}(1\text{e-}4)) \end{aligned} \quad (4.16)$$

Fitting the model with the MLE of parameters, verifying convergence, viewing the system and observation variance, checking the Hessian positive definite, and standard errors gives:

```
> modspdTrig <- buildspdTrig(fitspdTrig$par) ## Fitted model
> fitspdTrig$convergence ## Check Convergence
[ans] 0(TRUE)
> drop(W(modspdTrig)) ## System Variance
[ans] 9.167269
> drop(V(modspdTrig)) ## Observation Variance
[ans] 4.201806
> hs <- hessian(function(x) dlmLL(spd1, buildspdTrig(x)), fitspdTrig$par)
> all(eigen(hs, only.values = TRUE)$values > 0) ## Positive Definite?
[ans] TRUE
> aVar <- solve(hs) ## Asymptotic Variance / Covariance Matrix
> sqrt(diag(aVar)) ## Standard Errors
[ans] 0.7445461 1.1636410
```

4.3.3 KF Results: Traffic Speeds RW Model

The fitted model **modspd1** is:

$$\begin{aligned} \mathbf{G} &= [1] \\ \mathbf{W} &= [9.13] \\ \mathbf{F} &= [1] \\ \mathbf{V} &= [4.37] \end{aligned} \quad (4.17)$$

The Kalman filter recursive algorithm was applied to (4.17) and the results of the one-step-ahead forecasts are shown in Figure 16. The KF results are excellent and converge

within three intervals. The model is able to track the measurements exceptionally even in speed reductions of 35 mph occurring in a 15 minute period as shown at TI 665 to TI 668. The fact that the drop in speed occurred over 15-minutes is insignificant; the fact that it occurred over three observations without much error is. Another words, the sampling time is arbitrary compared to the intervals it occurred over.

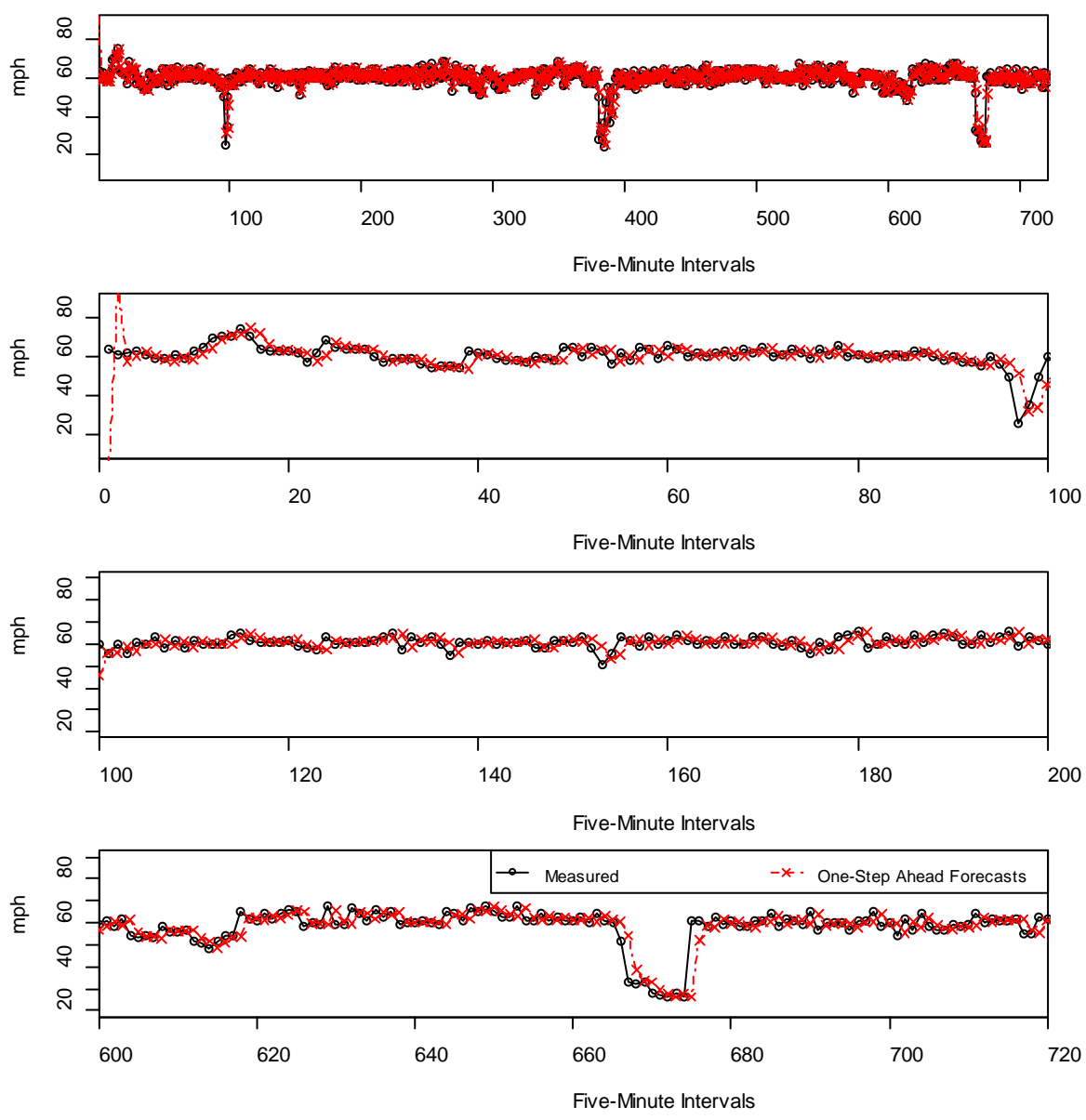


Figure 16. Kalman Filter Applied to Traffic Speed Random Walk with Noise Model

4.3.4 KF Results: Traffic Speeds RW with Seasonal Component Model

The fitted model **modspdseas** is:

$$\begin{aligned}
 \mathbf{G} &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \\
 \mathbf{W} &= \begin{bmatrix} 10.65 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
 \mathbf{F} &= [1 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] \\
 \mathbf{V} &= [1.48]
 \end{aligned} \tag{4.18}$$

The Kalman filter recursive algorithm was applied to (4.18) and the results of the one-step-ahead forecasts are shown in Figure 17. The data was converted into one-hour intervals, and although not representative of real-world, the goal was to capture any seasonal patterns that may have existed. Unexpectedly, the signal-to-noise ratio was the largest, thus the model had the greatest adaptive capabilities of the three speed models. The signal-to-noise ratios were similar for the RW, with 2.09, and the Fourier-form, with 2.29, models.

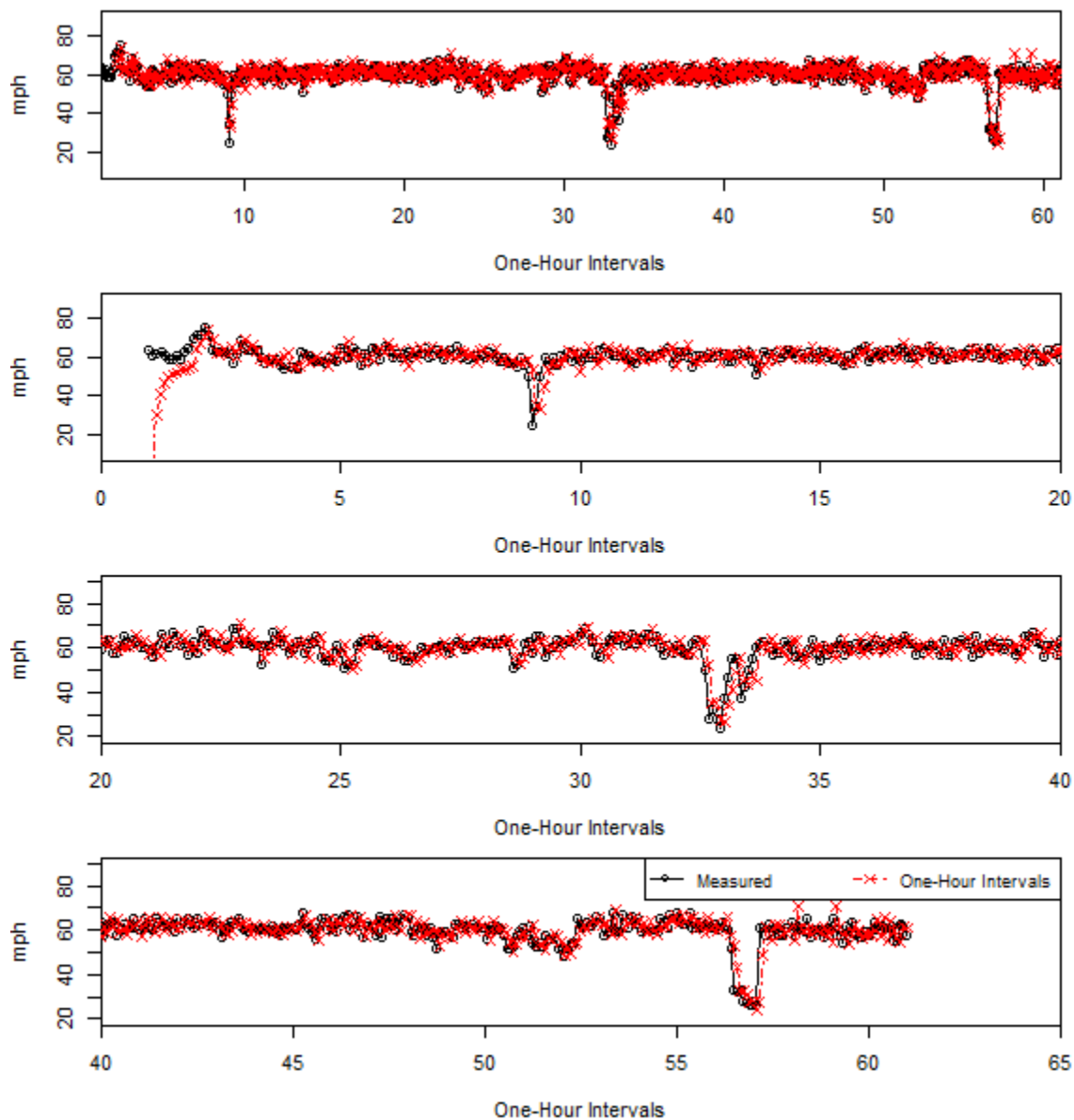


Figure 17. Kalman Filter Applied to Traffic Speed Random Walk with Seasonal Component Model

4.3.5 KF Results: Traffic Speeds RW with Fourier-Form Model

The fitted model **modspdTrig** is:

$$\begin{aligned}
 \mathbf{G} &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.87 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.5 & 0.87 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0.87 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.87 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2.8e-16 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 2.8e-16 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.5 & 0.87 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.87 & -0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.87 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.5 & -0.87 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \\
 \mathbf{W} &= \begin{bmatrix} 9.6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
 \mathbf{F} &= [1 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1] \\
 \mathbf{V} &= [4.20]
 \end{aligned} \tag{4.19}$$

The Kalman filter recursive algorithm was applied to (4.19) using five-minute interval data and the results of the one-step-ahead forecasts are shown in Figure 18.

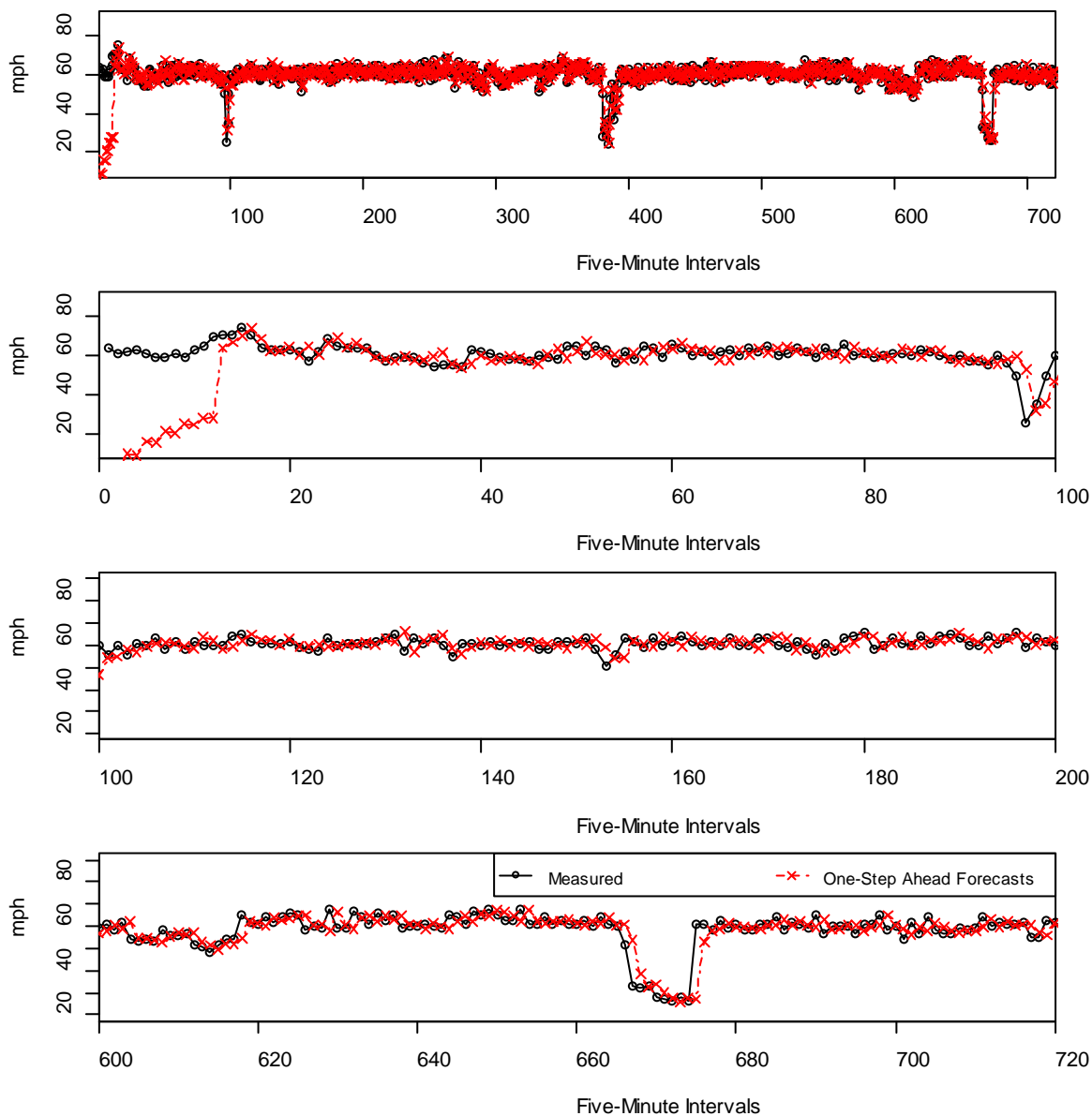


Figure 18. Kalman Filter Applied to Traffic Speeds Random Walk with Fourier-form Seasonal Component Model Five-Minute Frequency Data

4.4 Results Analysis

The DLMs constructed for the modeling of traffic volume all performed exceptionally well with Fourier-form seasonal component model having the smallest standard errors. Two performance indices were used to rank the models: the Root Mean Square Error of Prediction (RMSEP), calculated as:

$$RMSEP = \sqrt{\frac{1}{N} \times \sum_{k=1}^N \tilde{y}_k^2} \quad (4.20)$$

and the Mean Absolute Deviation (MAD):

$$MAD = \frac{1}{N} \sum_{k=1}^N |\tilde{y}_k| \quad (4.21)$$

where N is the number of observations, and \tilde{y}_k is the residual.

The performance statistics for the traffic volume DLMs can be found in Table 4.1.

Table 4.1. DLM Volume Models Performance statistics

Model	MAD	RMSEP
Random Walk with Noise	7.08	9.74
RW with Seasonal Component	7.64	10.43
RW with Fourier-form	7.59	10.91

As seen in Table 4.1, the RW with noise model performed better than the other two models. It should be noted that these statistics are not used for any other purposes than ranking the models among one another.

The performance statistics for the traffic speeds DLMs can be found in Table 4.2.

Table 4.2. DLM Speed Models Performance statistics

Model	MAD	RMSEP
Random Walk with Noise	3.00	3.19
RW with Seasonal Component	3.53	3.78
RW with Fourier-form	3.70	4.77

As seen in the above table, the RW model performed better than the other two models.

The **R** environment provided a platform to model data from traffic speeds and traffic volumes. The results from the DLM and KF models were used as the beginning point for the matrices used in MATLAB, described in the next section. The RW plus noise traffic and speed models were chosen for this task. The **R** file, **Section_4.2.R**, for the traffic volume DLMs and KF, can be found in Appendix C. The file, **Section_4.3.R**, contains the **R** code for the traffic speed DLMs and KF and can be found in Appendix D.

CHAPTER 5: PREDICTION & CONTROL INTEGRATION

This chapter combines, in MATLAB, the Kalman filter algorithm presented in Section 3.2 with the control logic presented in Section 3.5. The KFs role is to recursively forecast the traffic state variables to be used in the control scheme. The traffic state can be represented by variables that describe the state of the freeway system such as traffic flow, speed, density, or occupancy. The integration of the KF with the ramp meter control logic completes the ramp meter algorithmic scheme which is proactive to changes in freeway conditions by controlling a forecasted state.

Numerical implementation of the KF generates numerical errors, even when the optimal filter is utilized – e.g., due to round-off errors (Bucy & Joseph, 2005). Joseph (Bucy & Joseph, 2005) suggested a form for the error covariance update that avoids this, although, at the expense of computational burden, this form was used in our Kalman filtering routine. The Joseph-form error covariance estimate (*a posteriori*):

$$\mathbf{P}_{k|k} = (\mathbf{I} - \mathbf{K}_k \mathbf{F}_k) \mathbf{P}_{k|k-1} (\mathbf{I} - \mathbf{K}_k \mathbf{F}_k)^T + \mathbf{K}_k \mathbf{R}_k \mathbf{K}_k^T \quad (5.1)$$

where \mathbf{I} is an identity matrix with dimensions equal to the length of the observation matrix (or vector). Equation 5.1 was used in place of the posteriori covariance Equation 3.11 to avoid numerical errors associated with round-off.

5.1 Integration Methodology

The KF makes predictions based on the freeway system output (i.e., the state of the traffic) for each sections density ρ_i and the on-ramp queue lengths l_i . The queue lengths are estimated by the KF because they are used by the state feedback controller in the calculation of the metered flow u_i . Also, there is continuity of inputs and outputs between algorithms, meaning the output from the KF is feeding the input of the controller. The KF compares its prediction with the system outputs, obtained from roadway sensors, and the controller compares its error signal based on the difference in the KF output and reference signal. The integration of the KF and the control algorithm is represented in a block diagram in Figure 19.

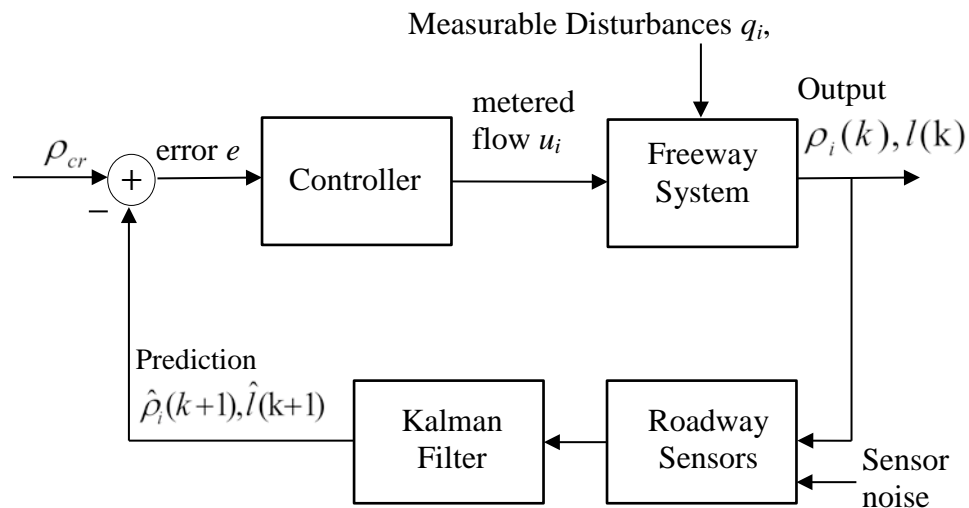


Figure 19. Kalman Filter and Control System Block Diagram

The error term for the controller is a function of the mainline density ρ and queue length l . However, since the reference input for the controller is the critical density ρ_{cr} only, the question may arise as to how the controller uses the KF's queue length prediction in the error calculation and control input. The answer is in the fact that the controller's

state feedback logic calculates the error as a function of the absolute value of $(\rho_{cr} - \rho_i)$ and the queue length l_i given by:

$$e(k) = w_1 |\rho_1(k) - \rho_{cr1}| + w_2 l_1(k) + w_3 |\rho_2(k) - \rho_{cr2}| + w_4 l_2(k) \quad (5.2)$$

where the queue length is calculated from the difference in measurable disturbances r_i and the controllers output u_i , plus the previous time steps queue length. Therefore, the “state” of the freeway system is used in the feedback loop to adjust the metering rate u_i . Clearly, the weighting factors determine the significance given to the mainline sections or ramps through (5.2).

The action taken by the controller to adjust the control output based on the error is proportional-derivative (PD), as described in Section 3.4. The controller’s block diagram for the PD action is shown in Figure 20.

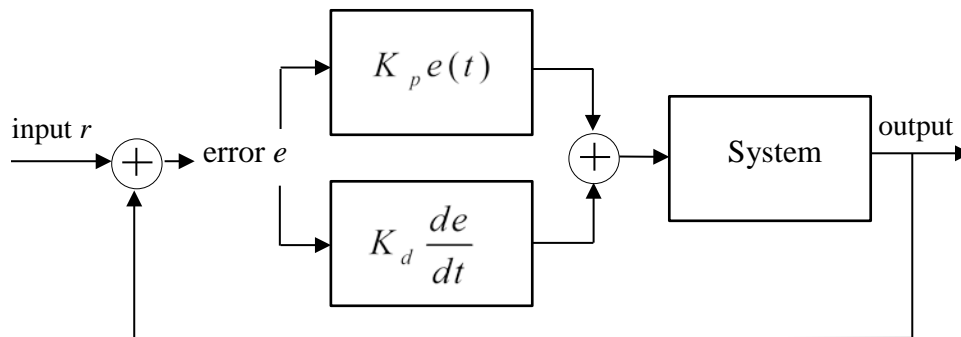


Figure 20. Proportional-Derivative Controller Action

The integration of the prediction algorithm and control logic can be thought of as the complete ramp metering algorithmic scheme. The complete ramp meter scheme includes the detectors that collect traffic data both on the mainline and the on-ramps, the ramp meter signal actuators, and the algorithmic scheme.

5.2 MATLAB Programming: Kalman Filter & Extended Kalman Filter

The observation vector consisted of density measurements from EB sites 5 and 6 that were calculated from flow (veh/hr) and speed (mph) measurements. The measurements recorded at site 5 were assumed to correspond to the density of section 1, containing the Eagle Road EB loop on-ramp (Ramp 1) as shown in Figure 21. The measurements recorded at site 6 were assumed to correspond to the density of section 2, containing the Eagle Road EB on-ramp (Ramp 2). These measurements can be collected in real-time with roadway detectors, shown below by the blue dashed lines.

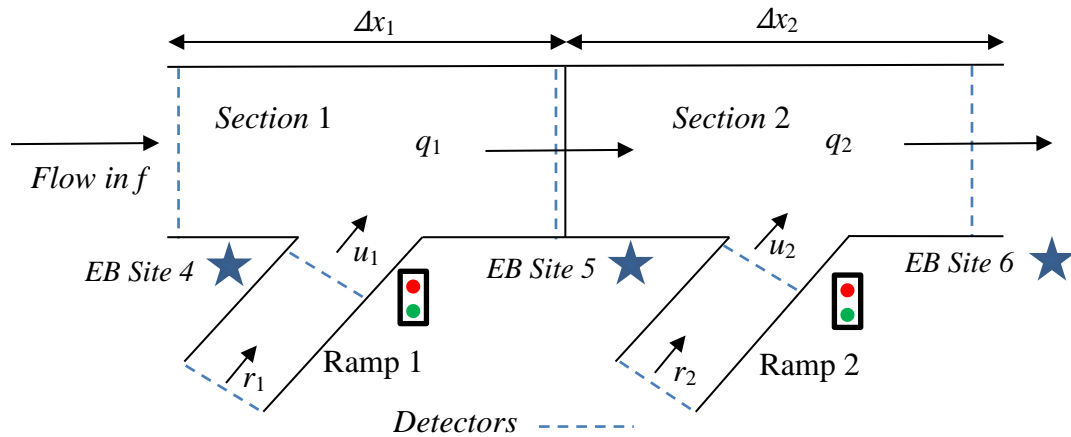


Figure 21. Linearized Freeway System with Radar Sensor's Locations

5.2.1 Kalman Filter

The discrete-time Kalman filter performs the recursive prediction-correction estimation. The methodology present in Sections 3.2, 4.2, and 4.3 were numerically programmed into MATLAB. A Kalman filter was conceptualized and programmed as a “dual-state” estimation algorithm, computing one-step ahead densities at Section 1 and Section 2 (as shown in Figure 21). One-step ahead refers to the next discrete time $k+1$

prediction in which some time interval, Δt , has lapsed. The discrete time step from k to $k+1$ is usually equal to the sampling time (T) where $kT = \Delta t$.

The dual-state Kalman filter matrices were programmed in MATLAB as:

$$\begin{aligned}\mathbf{G}_k &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ \mathbf{Q}_k &= \begin{bmatrix} 3.7330 & 0 \\ 0 & 5.6221 \end{bmatrix} \\ \mathbf{F}_k &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ \mathbf{R}_k &= \begin{bmatrix} 1.1034 & 0 \\ 0 & 2.8308 \end{bmatrix}\end{aligned}\tag{5.3}$$

where \mathbf{G}_k is the state design matrix, \mathbf{Q}_k is the variance of the state noise \mathbf{w}_k , \mathbf{F}_k is the observation matrix, and \mathbf{R}_k is the variance of the observation noise \mathbf{v}_k . The parameters of the model were fitted in \mathbf{R} using MLE with similar methodology as outlined in Section 4.2.1. The filter was initialized with the following state process matrix \mathbf{x}_0 and its error covariance \mathbf{P}_0 :

$$\begin{aligned}\mathbf{x}_0 &= \begin{bmatrix} 25 \\ 35 \end{bmatrix} \\ \mathbf{P}_0 &= \begin{bmatrix} 15 & 0 \\ 0 & 10 \end{bmatrix}\end{aligned}\tag{5.4}$$

The initial state and error matrices (5.4) were estimated from EB site 5 and site 6 radar data. In MATLAB, the Kalman filter recursions were programmed rather instinctively with a “for-loop”:

```

for ii=1:length(flow5)
(3.5)   xpred = G*x_hat;
        predrho(ii,:) = xpred(:,1)';
(3.6)   ppred = G*pplus*G'+Q;
(3.7)   y_hat = meas(ii,:)-F*xpred;
        resid(ii,:) = y_hat';
(3.9)   K = (ppred*F') / (F*ppred*F' + R);
(3.10)  x_hat = xpred + K*y_hat;
(5.1)   pplus = (eye(2) - K*F)*ppred*(eye(2) - K*F)' + K*R*K';
end

```

where the equation number in the left margin corresponds to those found herein.

To build a suitable KF, there are four tuning parameters that are determined prior to the filters implementation (Saha, Goswami, & Ghosh, 2013). These are the initial state estimate \mathbf{x}_0 and the three noise covariance matrices: the initial state estimation error covariance \mathbf{P}_0 , the state noise covariance \mathbf{Q}_k , and the measurement noise covariance \mathbf{R}_k . Since \mathbf{x}_0 and \mathbf{P}_0 change after the first iteration of the algorithm, the state and the measurement noise covariances \mathbf{Q}_k , \mathbf{R}_k are more critical in tuning. Subsequently, the covariance \mathbf{R}_k can be determined from knowledge of the sensors characteristics so the state noise covariance \mathbf{Q}_k is considered to be the most critical tuning parameter (Saha et al., 2013).

5.2.2 Extended Kalman Filter

The Kalman filter is only optimal under certain conditions. The process being estimated is assumed to be a linear dynamical system driven by stochastic processes in the presence of noise. The noise corrupting the system and its measurements are assumed to be Gaussian, white noise processes, and independent, or not correlated in

time. Most processes occurring in the real-world violate one or both of these assumptions; however, there are justifications in relaxing these assumptions. For example, if the Gaussian assumption is removed, the Kalman filter can still be the best minimum error variance filter (Maybeck, 1982).

Other variants of the Kalman filter exist such as the Extended Kalman filter (EKF), a non-linear estimator. An EKF was constructed where the nonlinear state and observation models were described by:

$$\mathbf{x}_k = f(\mathbf{x}_{k-1}) + \mathbf{w}_{k-1} \quad (5.5)$$

and

$$\mathbf{y}_k = h(\mathbf{x}_k) + \mathbf{v}_k \quad (5.6)$$

where $f(\cdot)$ and $h(\cdot)$ are nonlinear functions and \mathbf{v}_k and \mathbf{w}_k are Gaussian independent white noise sequences, as in the KF.

The EKF is similar to the KF except that the nonlinear functions $f(\cdot)$ and $h(\cdot)$ cannot be used directly for estimation, instead, their partial derivatives, known as Jacobian matrices are used. The state and observation matrices are defined by the partial derivative Jacobians (which are not given in vector notation to distinguish from previous KF equations) by:

$$F_{k-1} = \left. \frac{\partial f}{\partial \mathbf{x}} \right|_{\hat{\mathbf{x}}_{k-1|k-1}} \quad (5.7)$$

and

$$H_k = \left. \frac{\partial h}{\partial \mathbf{x}} \right|_{\hat{\mathbf{x}}_{k|k-1}} \quad (5.8)$$

The state estimate (*a priori*):

$$\hat{\mathbf{x}}_{k|k-1} = f(\hat{\mathbf{x}}_{k-1|k-1}) \quad (5.9)$$

Error covariance estimate (*a priori*):

$$\mathbf{P}_{k|k-1} = F_{k-1} \mathbf{P}_{k-1|k-1} F_{k-1}^T + \mathbf{Q}_k \quad (5.10)$$

where F_{k-1} is the Jacobian of function $f(\cdot)$ with respect to \mathbf{x} .

Measurement innovation (or residual):

$$\tilde{\mathbf{y}}_k = \mathbf{y}_k - h(\hat{\mathbf{x}}_{k|k-1}) \quad (5.11)$$

The innovation covariance:

$$\mathbf{S}_k = H_k \mathbf{P}_{k|k-1} H_k^T + \mathbf{R}_k \quad (5.12)$$

Kalman filter gain:

$$\mathbf{K}_k = \mathbf{P}_{k|k-1} H_k^T \mathbf{S}_k^{-1} \quad (5.13)$$

Updated state estimate (*a posteriori*):

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k \tilde{\mathbf{y}}_k \quad (5.14)$$

Updated error covariance estimate (*a posteriori*):

$$\mathbf{P}_{k|k} = \mathbf{P}_{k|k-1} - \mathbf{K}_k H_k \mathbf{P}_{k|k-1} \quad (5.15)$$

The EKF model were initialized with the following state, error covariance, and noise matrices in MATLAB:

$$\begin{aligned}
\hat{\mathbf{x}}_0 &= [15 \ 5 \ 15 \ 5]^T \\
\mathbf{P}_0 &= \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} \\
\mathbf{Q}_k &= \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 15 & 0 \\ 0 & 0 & 0 & 6 \end{bmatrix} \\
\mathbf{R}_k &= \begin{bmatrix} 7 & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix}
\end{aligned} \tag{5.16}$$

The first and third parameters for $\hat{\mathbf{x}}$ were approximated from the KF estimated densities.

The second and fourth parameters for $\hat{\mathbf{x}}$, representing queue length, were necessary for the algorithm to converge and considered a reasonable initial queue length. To determine the nonlinear function $f(\cdot)$ for the state process, a model was determined from

Greenshields et al. (1935) model for speed and the fundamental traffic Equation (3.12):

$$q_{1,2} = v f_{1,2} \hat{\mathbf{x}}_{1,3} \left(1 - \frac{\hat{\mathbf{x}}_{1,3}}{\text{rhom}} \right) \tag{5.17}$$

where $\hat{\mathbf{x}}_{1,3}$ is the section 1 and section 3 density, from the predicted state vector $\hat{\mathbf{x}}$; $v f_{1,2}$ and rhom as previously defined. The nonlinear function $f(\cdot)$ is achieved by substituting $q_{1,2}$ (5.17) into (3.22), and substituting the state vector $\hat{\mathbf{x}}_{2,4}$ in for queue lengths $l_{1,2}$ into (3.22), which gives:

$$f = \begin{bmatrix} \hat{x}_1 + \frac{T}{\Delta x_1} \left(flow_1 - v f_1 \hat{x}_1 \left(1 - \frac{\hat{x}_1}{r \text{ hom}} \right) \right) \\ \hat{x}_2 + T (ramp_1 - u_1) \\ \hat{x}_3 + \frac{T}{\Delta x_2} \left(v f_2 \hat{x}_1 \left(\frac{\hat{x}_1}{r \text{ hom} - 1} \right) - v f_2 \hat{x}_3 \left(1 - \frac{\hat{x}_3}{r \text{ hom}} \right) \right) \\ \hat{x}_4 + T (ramp_2 - u_2) \end{bmatrix} \tag{5.18}$$

where $flow_1$ is the flow f represented in Equation (3.22) with the Jacobian for the state process (5.18) given by:

$$F = \begin{bmatrix} \frac{T\left(vf_1\left(\frac{\hat{x}_1}{\text{rhom}-1}\right)+vf_1\left(\frac{\hat{x}_1}{\text{rhom}}\right)\right)}{L1} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{T\left(vf_1\left(\frac{\hat{x}_1}{\text{rhom}-1}\right)+vf_1\left(\frac{\hat{x}_1}{\text{rhom}}\right)\right)}{L2} & 0 & \frac{T\left(vf_2\left(\frac{\hat{x}_2}{\text{rhom}-1}\right)+vf_2\left(\frac{\hat{x}_2}{\text{rhom}}\right)\right)}{L2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (5.19)$$

and the Jacobian of the measurement:

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (5.20)$$

As in the KF, the EKFs \mathbf{Q} and \mathbf{R} matrices determines to a large extent the behavior of the EKF. Generally, the higher the standard deviation (SD) of the measurement noise compared to the state noise, the lower the filter gain. A lower filter gain results the measurements $\mathbf{y}(k)$ having a lower impact of on $\hat{\mathbf{x}}_{k|k-1}$ and vice versa (Wang & Papageorgiou, 2005).

5.3 MATLAB Programming: Ramp Meter Control Files

The control logic presented in Section 3.5 was coded into MATLAB.

Obviously, traffic jams are unavoidable when traffic exceeds the physical limitations of the freeway system or external conditions, such as weather or incidents play a role.

In this study, the radar data was collected by lane. However, a general case was established by considering Lane 1 data only, the right-most lane in to which on-ramps merge, or to which they are adjacent to. This was a reasonable starting point because it was assumed most interactions with the on-ramp and freeway section take place at the interface between the merge or where interactions occurring between them, such as weaving and merging, are facilitated through.

The ramp meter control files were created in MATLAB as separate m-files (MATLABs file-type). The main file, **rampmeter_runfile.m**, contains the user inputs: gain regulator parameters, weighting factors, any other known parameters, and initial values. This is the main file that “calls” the other files that contain functions: **kalman_pred.m**, **ramp1_meter.m**, and **ramp2_meter.m**. Functions in MATLAB are custom programs that accept user inputs and return outputs, created by the user to perform a specific task. The function **kalman_pred.m** computes the forecasted system dynamics for the section density $\rho_i(k+1)$ and queue length $l_i(k+1)$. **Ramp1_meter.m** and **ramp2_meter.m** are the on-ramp metering control logic files that controls the release of vehicles to the mainline freeway. The main file also loads the data containing the radar measurements at each section (analogous to inductive loop data). The data file contained measurements for traffic flows f and q_i , ramp 1 and 2 demand volumes r_1 r_2 , average speeds, and the calculated density for sections 1 and 2.

The feedback gain parameters, mainline and ramp weighting factors, and other known parameters are shown in Tables 5.1 through 5.3. The parameters L1, L2, RL1, and RL2 of Table 5.1 are measurable parameters. The remaining parameters were determined from the fundamental diagrams of the measured radar data, provided in Appendix E Figure 42 and Figure 43. Because of relatively few observations in the congested zone, lanes 1 through 4 were included in those plots.

Table 5.1. Known Parameters for Ramp Meter Run-File

L1	0.298	Length of Freeway Section 1 (mi)
L2	0.391	Length of Freeway Section 2 (mi)
RL1	0.38	Length of Ramp 1 EB loop
RL2	0.29	Length of Ramp 2 EB On-Ramp
rhom	110	Jam Density (veh/mi)
rhoc1	33	Critical Density Section 1 (veh/mi)
rhoc2	41	Critical Density Section 2 (veh/mi)
vf1	75	Free-Flow Speed Section 1 (mph)
vf2	62	Free-Flow Speed Section 2 (mph)
T	5/60	Time-Step (hr)

Table 5.2. Initial Regulator Gains for Feedback Ramp Meter Control

dccgain	0.40	Decoupled Gain
ccgain	0.50	Coupled Gain
alpha1	0.65	Distribution Factor Ramp 1
alpha2	0.35	Distribution Factor Ramp 1

Table 5.3. Weighting Factors for Mainline and Ramp Sections

w1	0.35	Mainline Section 1
w2	0.65	Ramp 1 EB Loop
w3	0.35	Mainline Section 2
w4	0.65	Ramp 2 EB On-Ramp

`rampmeter_runfile.m`, `kalman_pred.m`, `ramp1_meter.m`, and `ramp2_meter.m` files can be found in the Appendix A: MATLAB Ramp Meter Control M-Files.

5.4 Simulation: KF & Decoupled Feedback Control Integration

Simulations were run on the decoupled and coupled control logic. The initial parameters for the decoupled control scheme are shown in Tables 5.1 through 5.3. The KF one-step-ahead predictions and measured sensor data for section 1 Time Interval (TI) 0 – 500 (where a TI is five-minutes) are shown in Figure 22; TI 500 – 1000 are shown in Figure 23. The KF one-step-ahead predictions and measured sensor data for section 2, TI 1000 – 1500, are shown in Figure 24; TI 1500 – 2000 are shown in Figure 25. The KF had excellent results and was able to track the density observations throughout the length of the simulation.

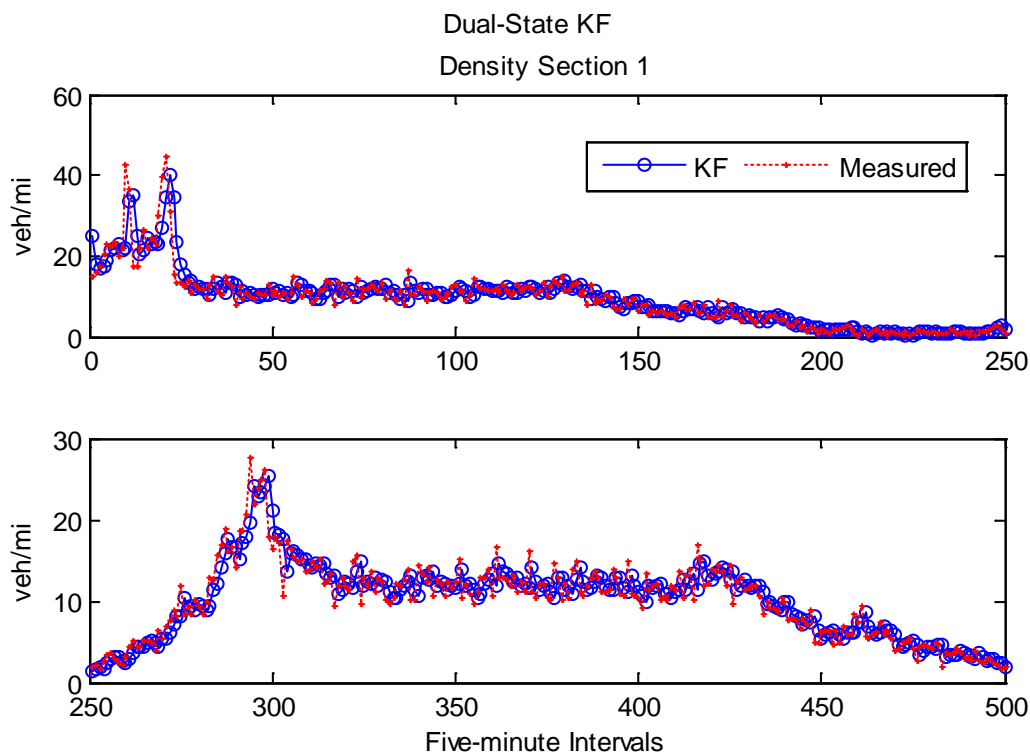


Figure 22. Kalman Filter Section 1 Density Predictions TI 0–500

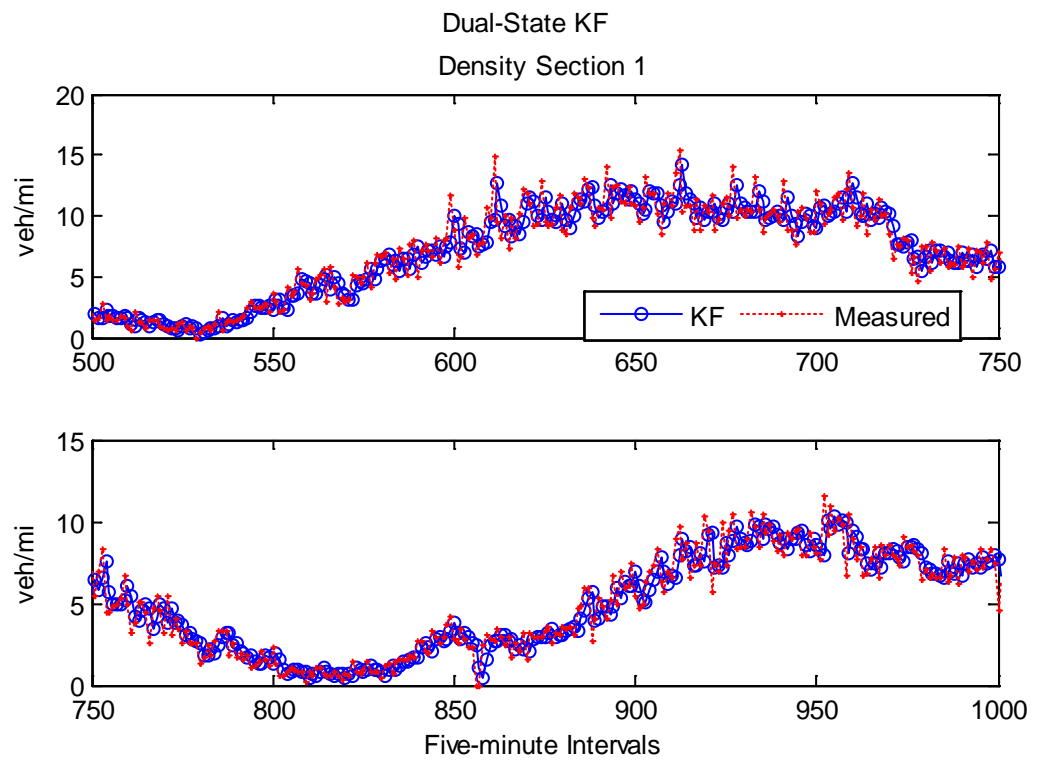


Figure 23. Kalman Filter Section 1 Density Prediction TI 500–1000

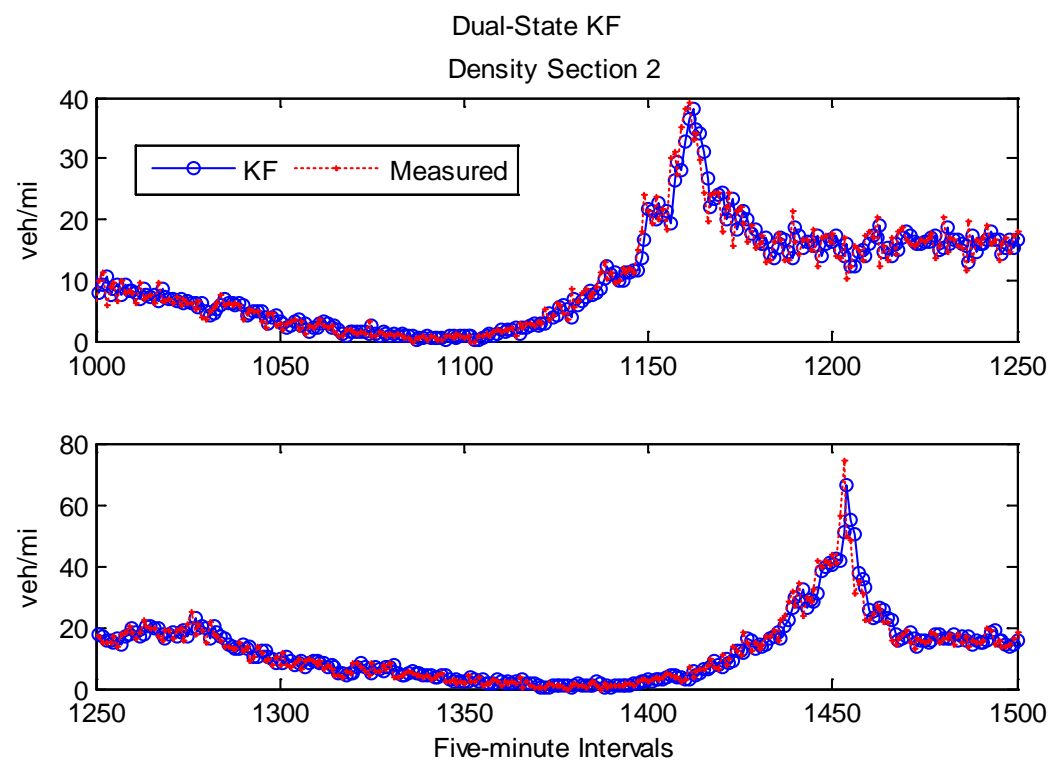


Figure 24. Kalman Filter Section 2 Density Prediction TI 1000–1500

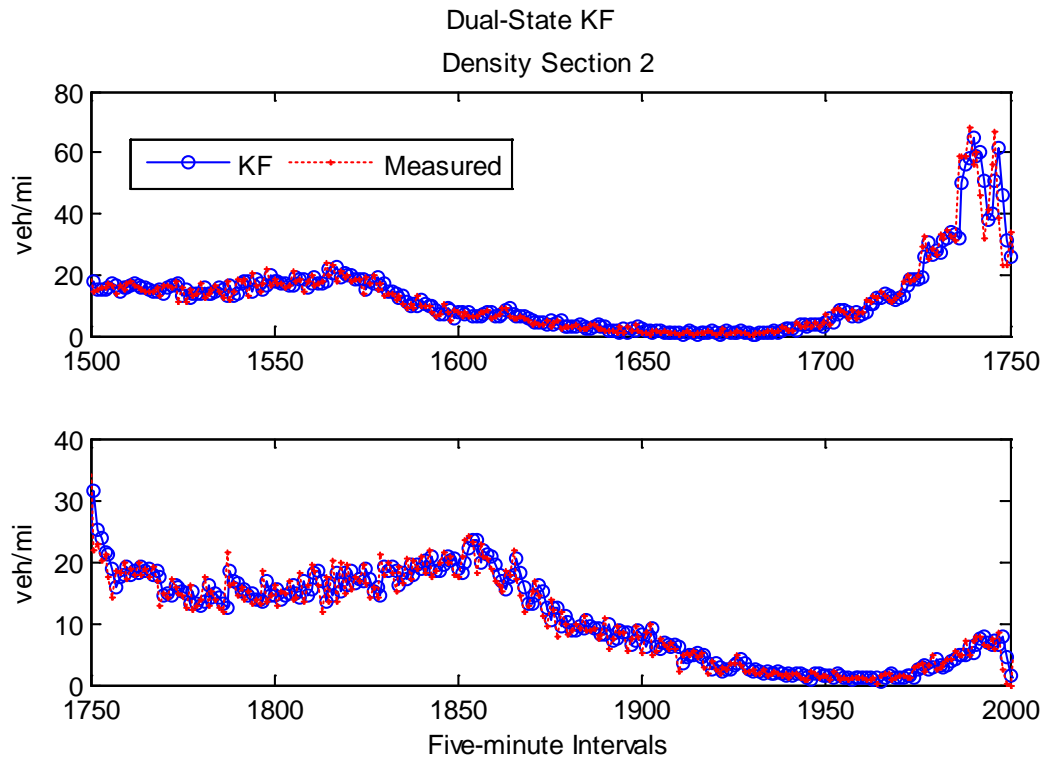


Figure 25. Kalman Filter Section 2 Density Prediction TI 1500–2000

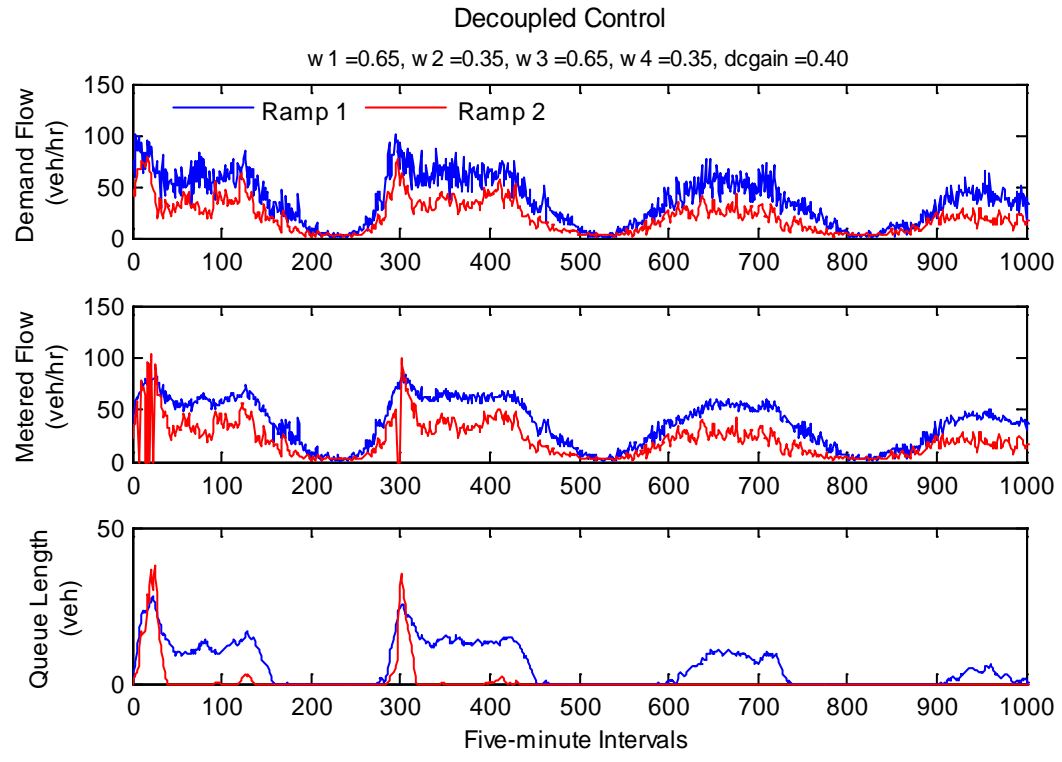


Figure 26. Decoupled Controls: Ramp Demand, Metered Flow, & Queue Length TI 0–1000

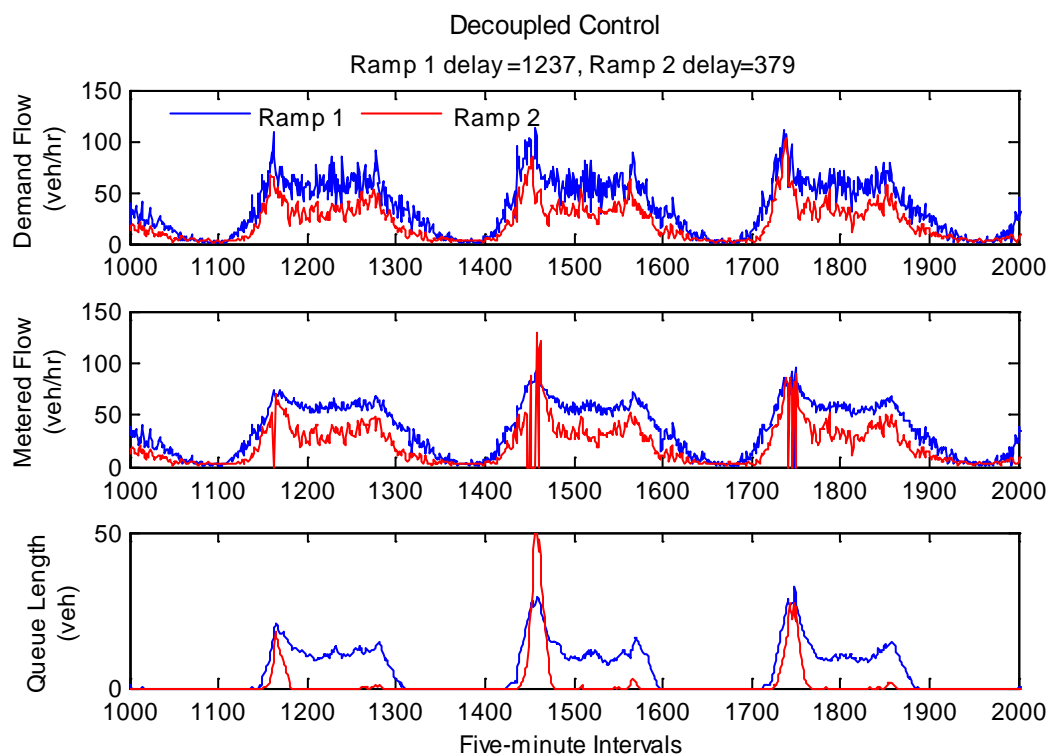


Figure 27. Decoupled Controls: Ramp Demand, Metered Flow, & Queue Length TI 1000–2000

The initial results with the decoupled gain $dgain$ set equal to 0.40 are shown in Figure 26 and Figure 27. Extended queue lengths began forming at approximately TI 0, TI 295, TI 600, TI 1170, TI 1420, and TI 1720, taking approximately 200 TIs to disperse. These periods all correspond to morning rush hour.

A performance metric named “Ramp 1, Ramp 2 delay,” displayed at the top of Figure 27, was developed to assess the controller performance. This metric is simply the number of observation periods that a queue was present. Since there is no way to determine timing of individual cars, details of impacts due to queues is impossible in this setting to calculate, without microsimulation. Ramp 1 experienced more than 1237 of the 2000 total TIs with a queue present. Ramp 2 experienced 379 of the total 2000 TIs with a queue present. Equally undesirable is the time the controller took to disperse the queue.

Although, nothing more than inference can be made, the number of time intervals with a queue can be viewed as a negative performance metric for comparing different controller configurations.

The residuals from the KF can be seen in Figure 28 and displays the root mean square error of the prediction (RMSEP), calculated as:

$$\sqrt{\frac{1}{N} \times \sum |\tilde{y}_k - \hat{x}_{k|k-1}|^2} \quad (5.21)$$

where N is the number of observations.

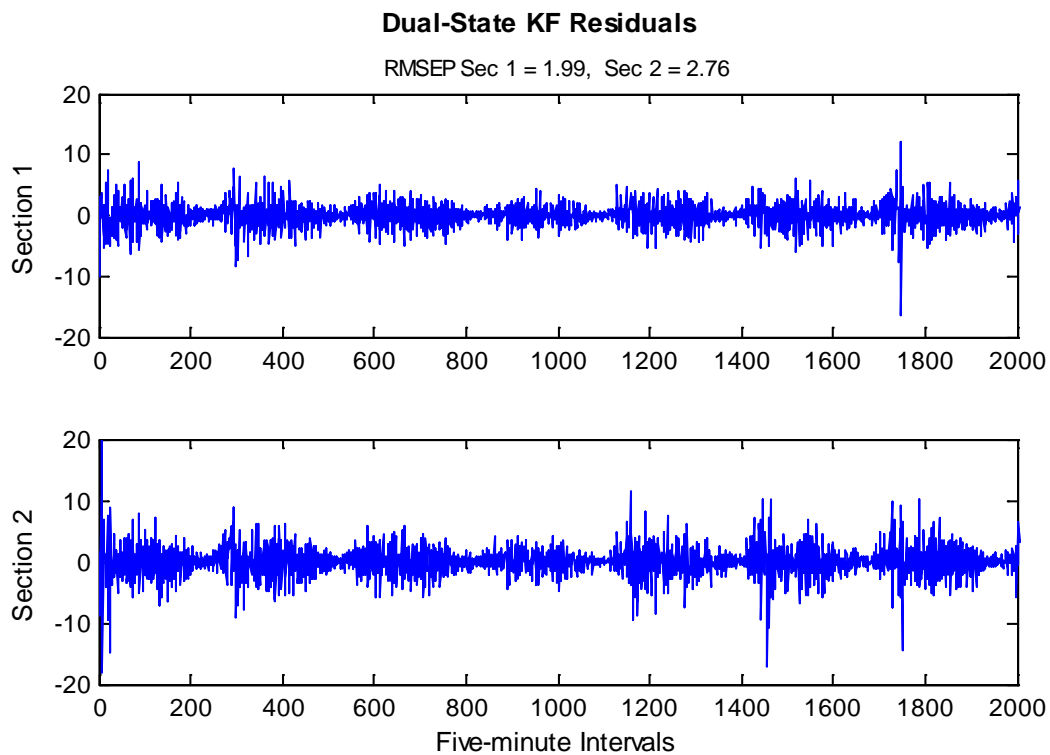


Figure 28. Kalman Filter Section 1 & 2 Residuals

The RMSEP for section 1 was 1.99 (veh/mi) and 2.76 (veh/mi) for section 2. A performance metrics, RMSEP, was used for criteria for evaluating the tested models.

What that means is the RMSEP was used only as an indicator of performance of similar models used in this study. The RMSEP should therefore be viewed as a tuning parameter. The RMSEP were consequently deemed acceptable with the 5-minute resolution data.

5.4.1 Decoupled Control Testing

Because the decoupled setting treats the ramp controllers as isolated controllers, the ramp demand was staggered so that a peak flow would occur at separate times for the ramps. They can be viewed as two independent on-ramps with their objectives only sought locally. The ramp 2 demand flows were shifted by approximately 6.7-hours (or 80 TI) to test the controller's performance when peak demands occurred at different periods.

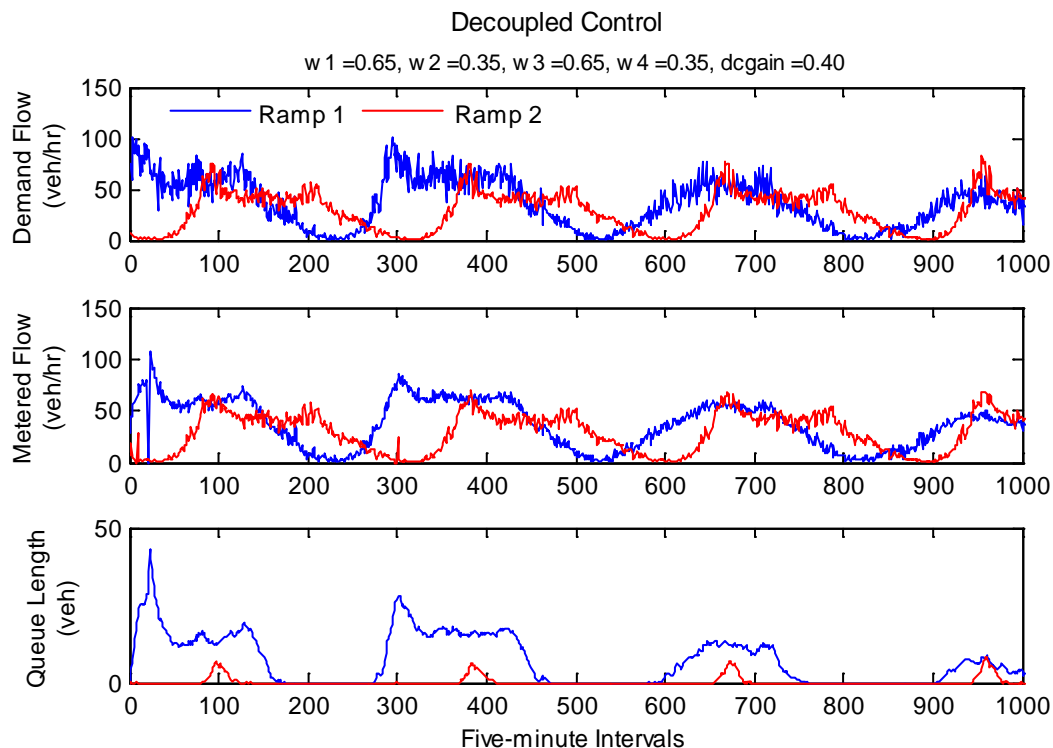


Figure 29. Decoupled Results TI 0–1000 for the Shifted Ramp 2 Demands

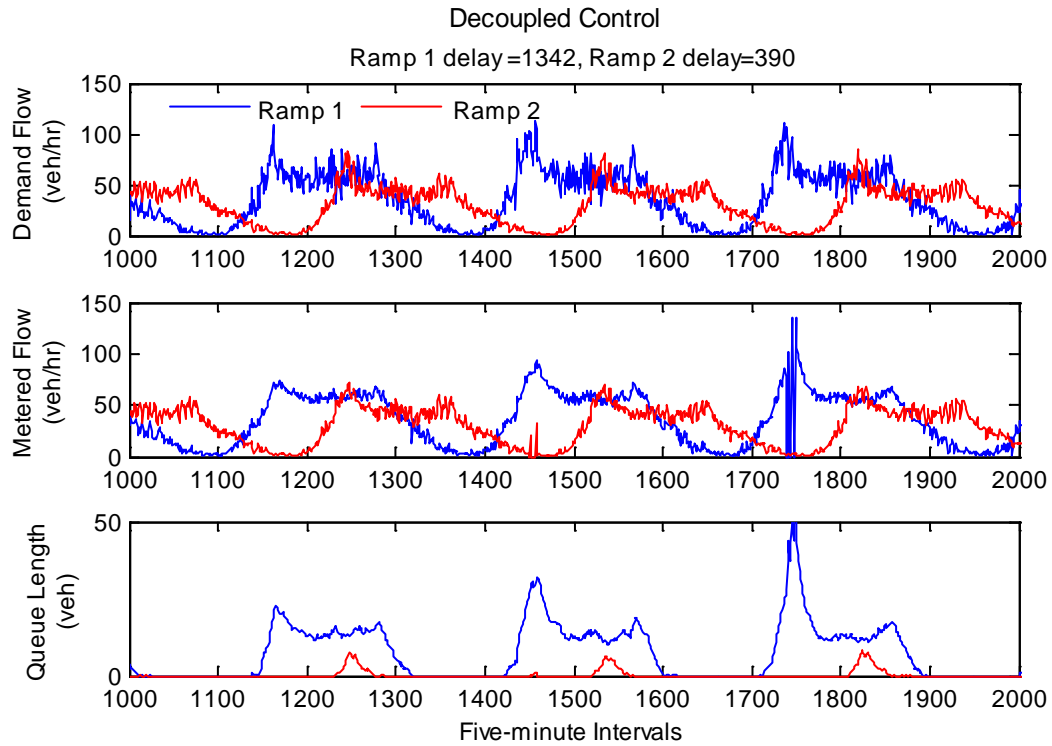


Figure 30. Decoupled Results TI 1000–2000 for the Shifted Ramp 2 Demands

The results of the shifted ramp 2 demand are shown in Figure 29 and Figure 30. Ramp 1 experienced less than 100 more periods with a queue present while ramp 2 experienced only 2 additional periods with a queue.

5.5 Simulation: KF & Coupled Feedback Control Integration

The integrated, coupled-control scheme was tested with the parameters given in Table 5.1 through Table 5.3. The distribution factors were 0.65 for the Loop On-Ramp and 0.35 for the EB On-Ramp. The values were chosen based on Kachroo and Ozbay, (2003).

The plots for the ramp demands r_i (veh/hr), metered flow u_i (veh/hr), and queue lengths l_i (veh) for the Loop On-Ramp and EB On-Ramp are shown in Figure 31 and Figure 32.

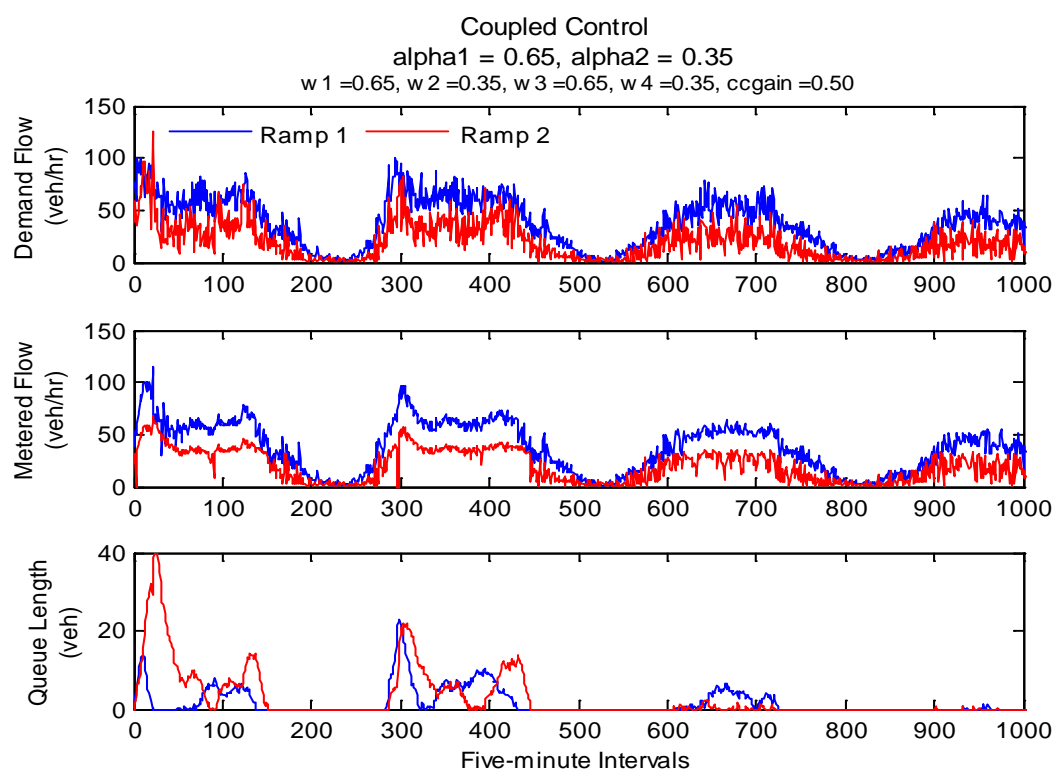


Figure 31. Coupled Controls: Ramp Demand, Metered Ramp Flow, & Queue Length TI 0–1000

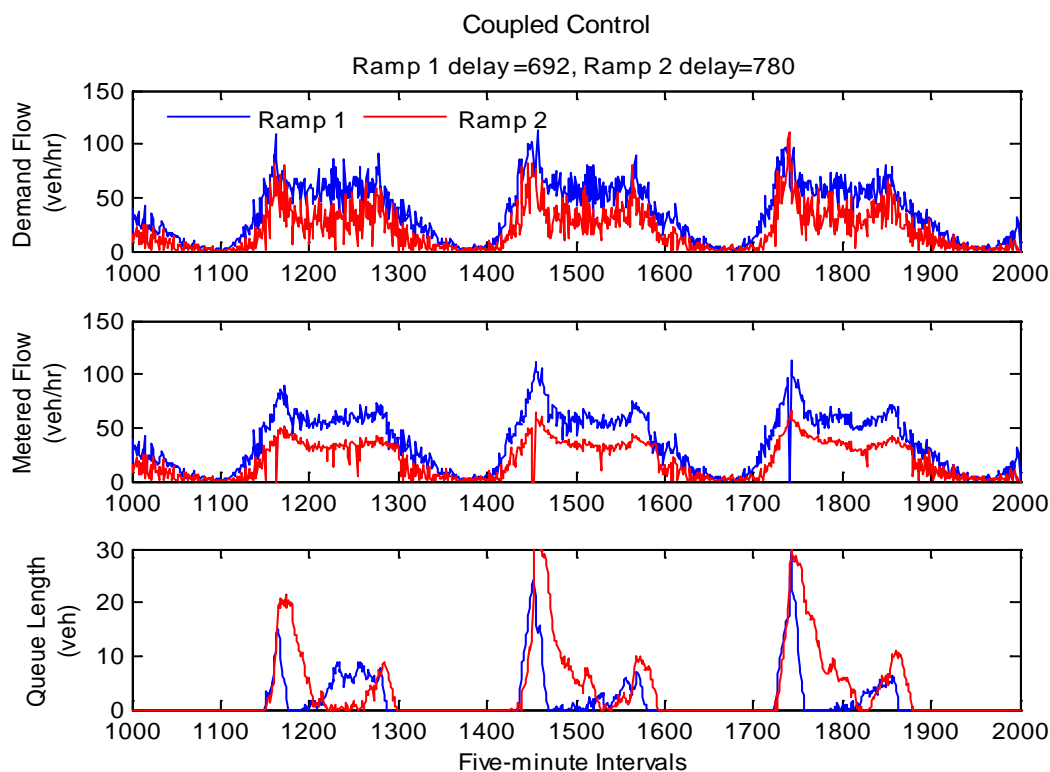


Figure 32. Coupled Controls: Ramp Demands, Metered Ramp Flow, & Queue Length TI 1000–2000

As seen in Figure 31 and Figure 32, the metered flow has much less turbulence than the unmetered ramp demand. On-ramp queue began forming at approximately TI 1, TI 300, TI 600, TI 1170, TI 1450, and TI 1750. The time to dissipate the queues were about 200 five-minute periods in all cases. These periods all correspond to morning rush hour.

The above plots reveal that EB on-ramp has periods where its queue formation exhibits extreme maximum peaks. The parameter w_4 was adjusted to 0.45 to give more weight to the on-ramp queue and the gain $ccgain$ was increased to 0.60; the results can be seen in Figure 33 and Figure 34.

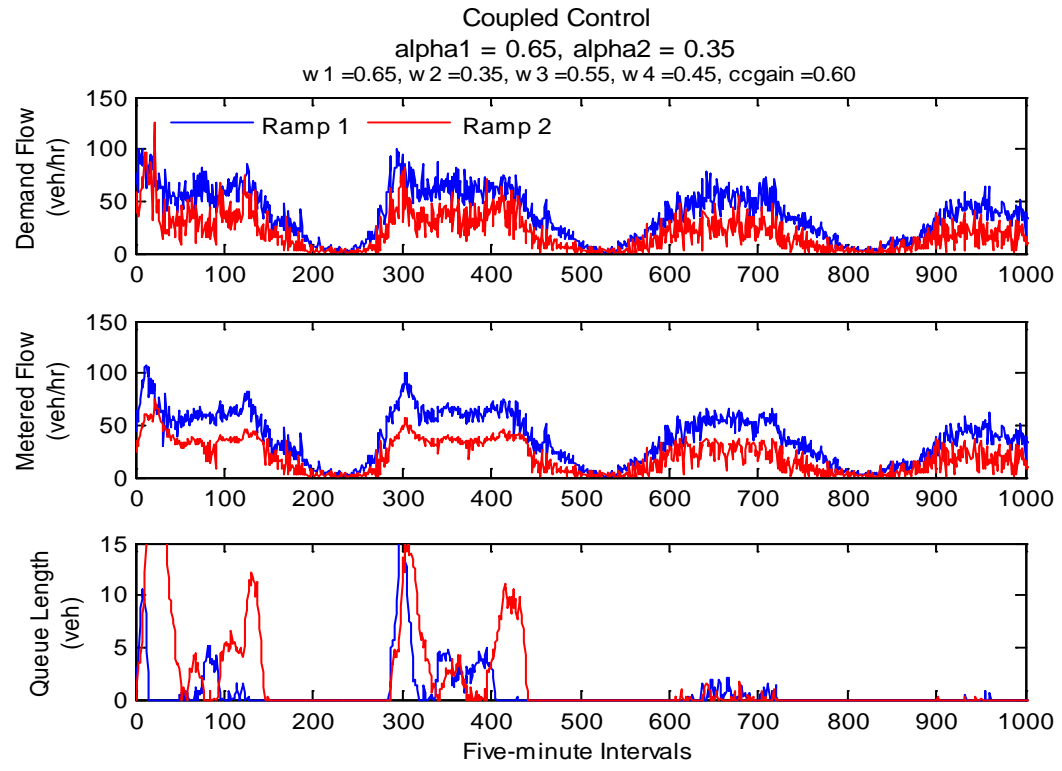


Figure 33. Coupled Controls: Adjusted Ramp 2 and Gain Parameters TI 0–1000

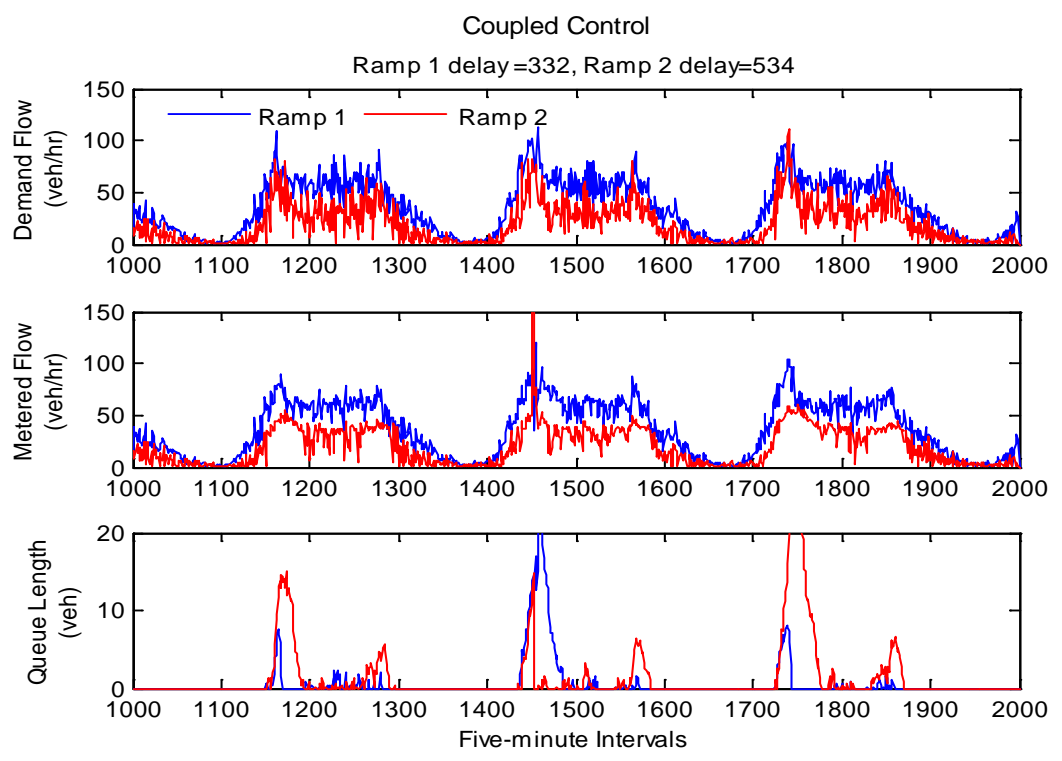


Figure 34. Coupled Controls: Adjusted Ramp 2 and Gain Parameters TI 1000–2000

As seen in the above figures, the extreme peaks of ramp 2's queue was reduced as well as the periods a queue was present on ramp 1. The above weighting parameter settings were repeated with an increased *ccgain* of 0.95; the results are below in Figure 35 and Figure 36.

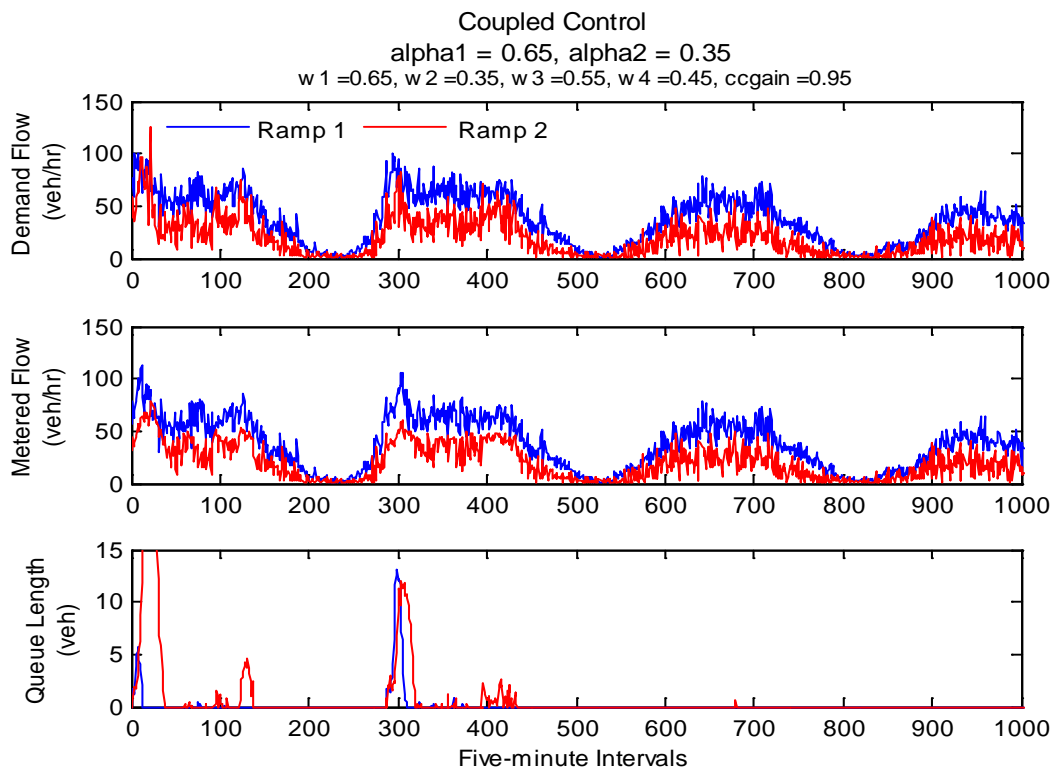


Figure 35. Coupled Controls: Adjusted Gain Parameters TI 0–1000

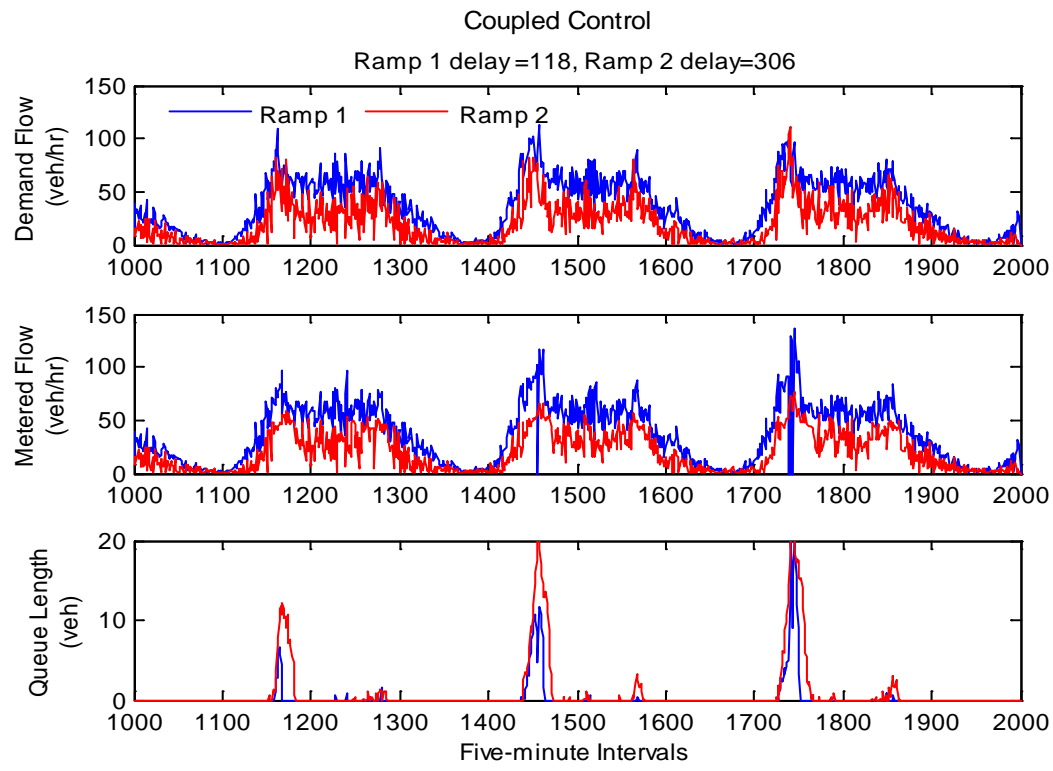


Figure 36. Coupled Controls: Adjusted Gain Parameters TI 1000–2000

This control configuration had excellent results with ramp 1 experiencing 118 observation periods with a queue present while ramp 2 experienced 306 observation periods with a queue present. The overall benefits of increasing the gain and establishing the magnitude will need to be tested in microsimulation.

5.6 Simulation: Extended KF Results

The EKF one-step-ahead predictions and measured sensor data for section 1 Time Interval (TI) 0 – 1000 (where a TI is five-minutes) are shown in Figure 37; TI 1000 – 2000 are shown in Figure 38. The EKF one-step-ahead predictions and measured sensor data for section 2, TI 0 – 1000, are shown in Figure 39; TI 1000 – 2000 are shown in Figure 40. The EKF had excellent results and was able to track the density observations throughout the length of the simulation. The residuals and RMSEP for sections 1 and 2 can be seen in Figure 41.

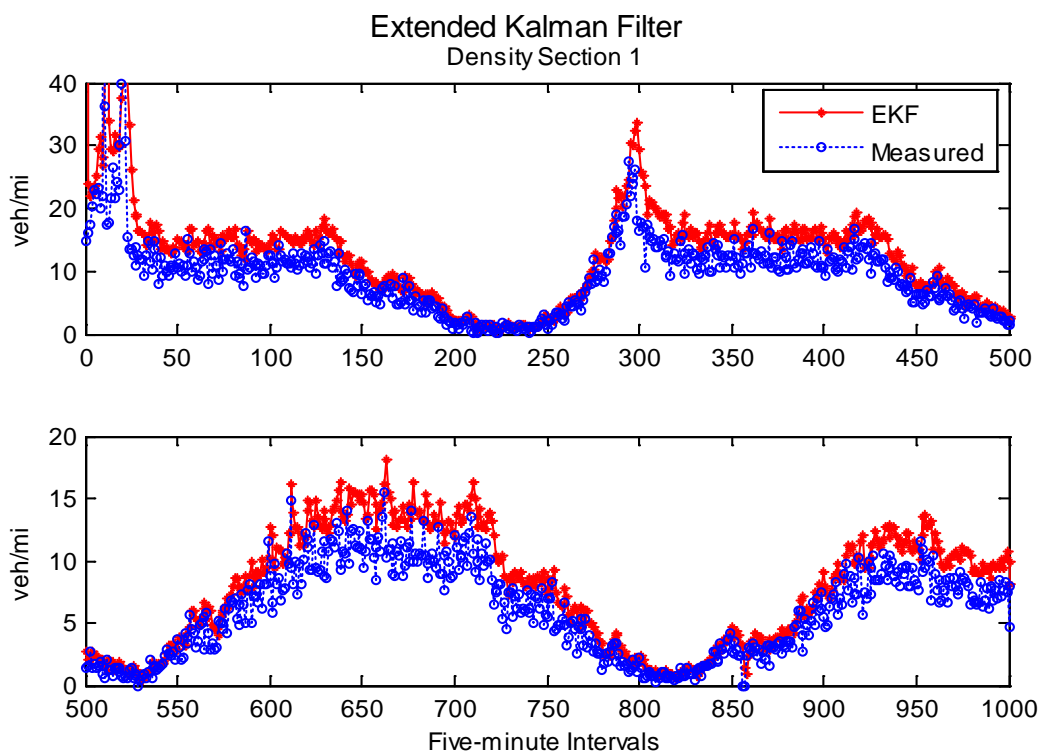


Figure 37. Extended Kalman Filter Prediction Section 1 Density TI 0–1000

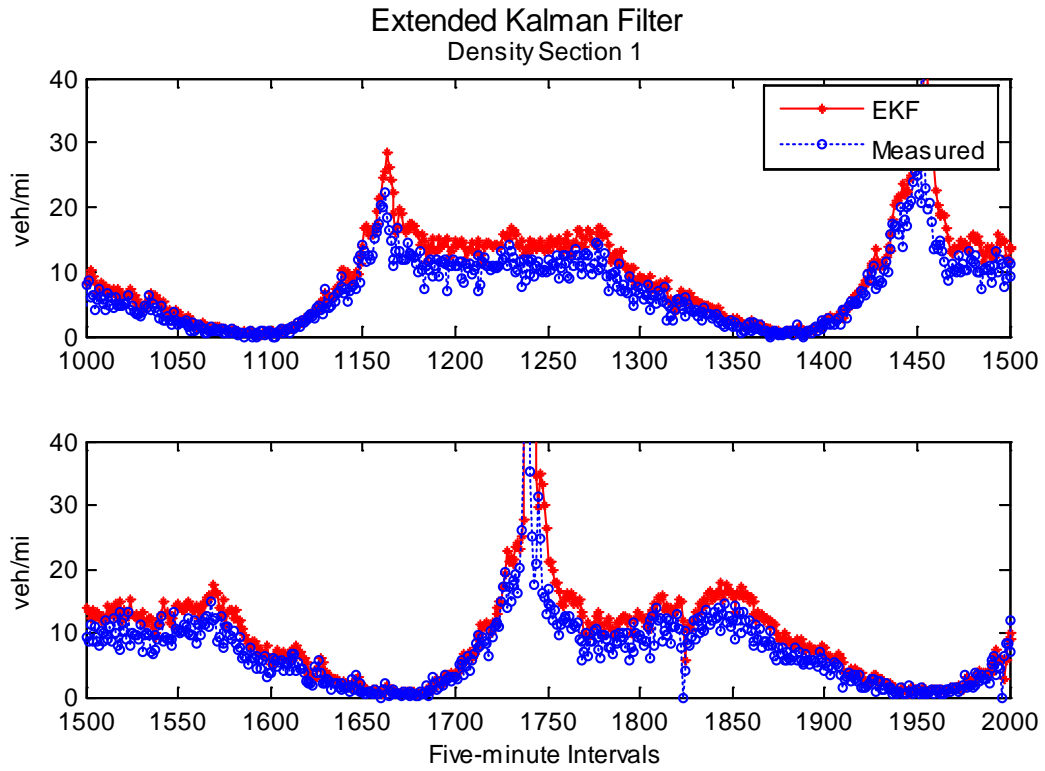


Figure 38. Extended Kalman Filter Prediction Section 1 Density TI 1000–2000

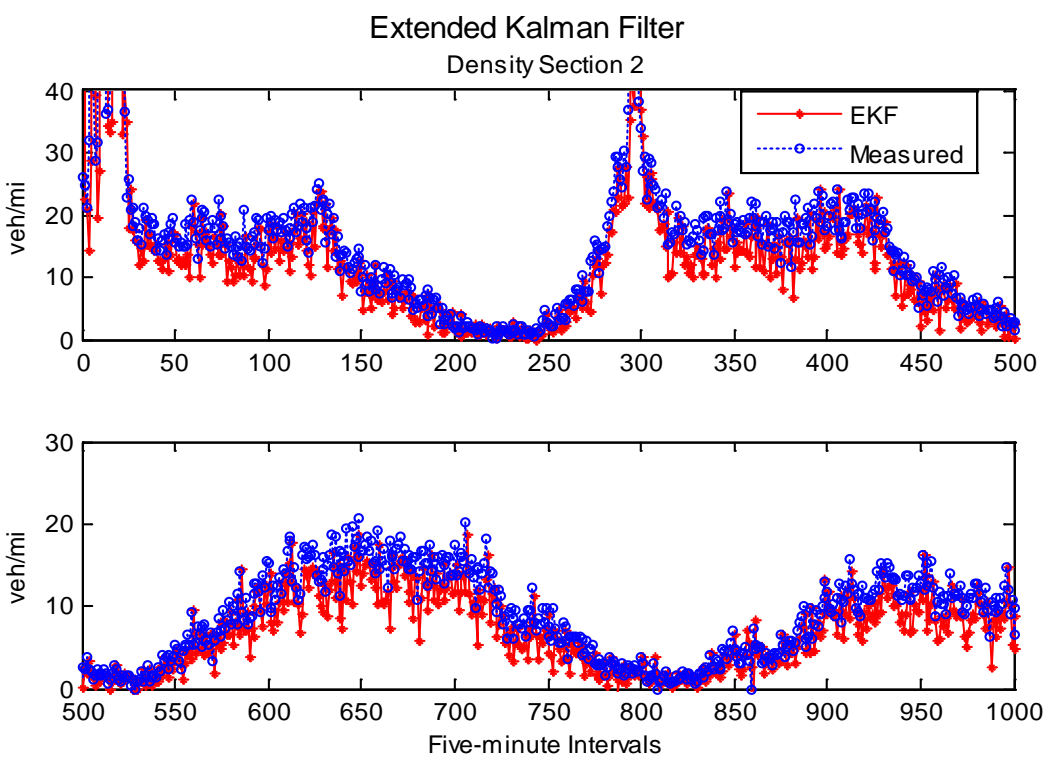


Figure 39. Extended Kalman Filter Prediction Section 2 Density TI 0–1000

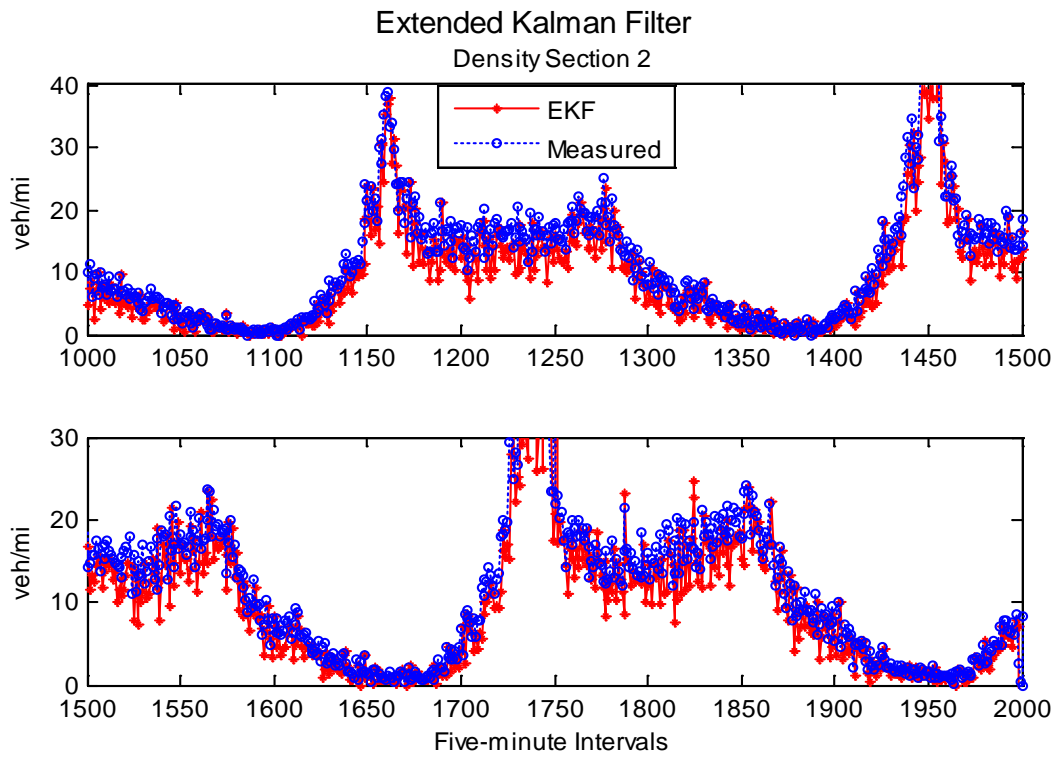


Figure 40. Extended Kalman Filter Prediction Section 2 Density TI 1000–2000

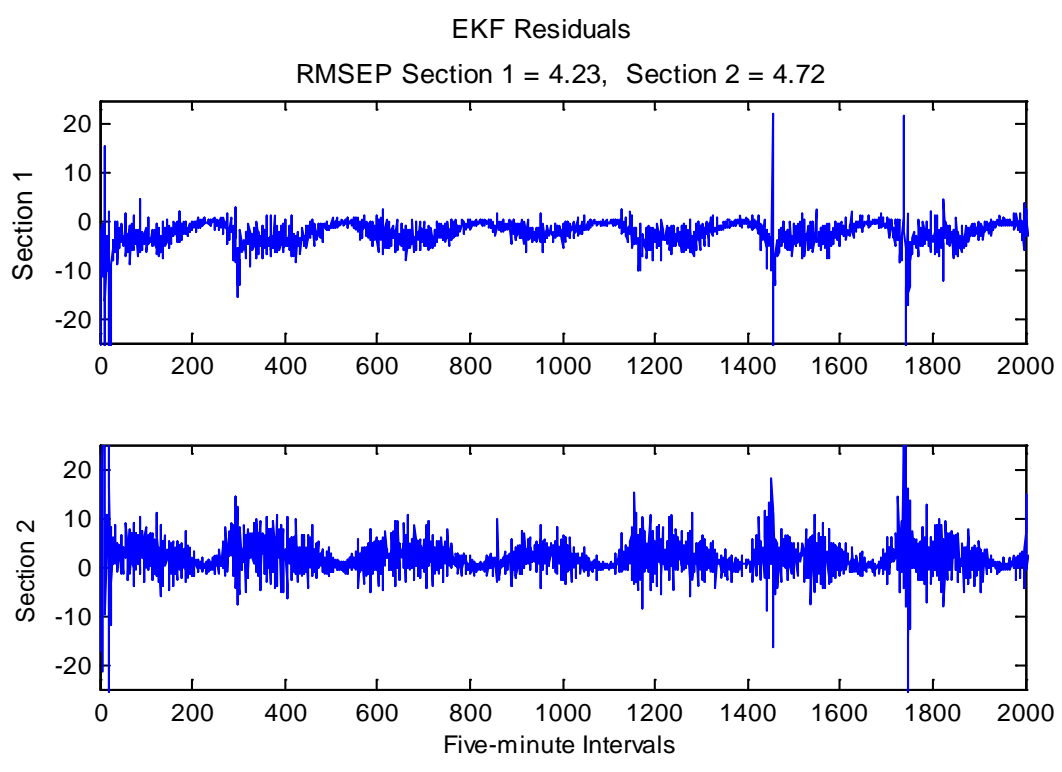


Figure 41. Extended Kalman Filter Prediction Residuals Sections 1 & 2

5.7 Results Analysis

The KF performed exceptionally well with low RMSEP for sections 1 and 2 of 2.46 (veh/mi) and 3.26 (veh/mi), respectively. Surprisingly, given the high non-linearity of the traffic data, it characterized the data well. As shown in the plots for the KF and EKF residuals, the errors are not independent. This is breaking an assumption with the Kalman filter. However, we are not claiming this to be the optimal solution. The noise is most likely correlated as seen in the residuals plots. The noise may not be Gaussian; however, this was less of a concern because the filter was not designed as the optimal solution.

For the main task of prediction, the filter performed satisfactorily, as the linear KF may still provide the best linear prediction. The EKF is known as being a difficult filter to tune and this case was no exception. Changes in the values for the noise or state covariance had significant effects on the performance of the model. The initial values for the state and state covariance error *a priori* were also particularly sensitive to the parameters, more so than the KF.

The results for the KF and EKF RMSEP are shown in Table 5.3.

Table 5.3. KF & EKF Root Mean Square Error of Prediction

Model	Section 1	Section 2	Total
Kalman Filter	1.99	2.76	4.75
Extended Kalman Filter	4.23	4.72	8.95

The results for the ramp 1 and ramp 2 delays for the most promising decoupled and coupled control schemes are shown in Table 5.4. The coupled control scheme was

superior to the decoupled; however, simulation and optimization is needed to determine the optimal settings for the weighting functions and control gain. In a traffic microsimulation, the stochastic nature of freeway traffic can be simulated, allowing for the results of a parameter's adjustment to be observed.

Table 5.4. Decoupled & Coupled Ramp Delays

Model	Ramp 1 Delay	Ramp 2 Delay	Total
Decoupled	1237	379	1616
Coupled	118	306	424

In either the decoupled or coupled setting, our metering control logic needs to be converted to a green-phase duration time for the ramp meter signal. The equation to convert the metering rate $u_i(k)$ to a green phase g is:

$$g = \frac{u_i(k)}{u_{sat}} C \quad (5.22)$$

where C is the cycle length (sec) and u_{sat} is the saturated ramp flow (veh/hr).

When either controller's distribution factor in the coupled setting is more than 0.65, the other ramp experiences queue growth at a much faster rate.

CHAPTER 6: CONCLUSIONS

With the rapid growth of the Treasure Valley and the distribution of housing and employment centers, congestion management will be vital as there is an increase in vehicle miles traveled. An increase in freeway capacity can result in congestion levels that quickly become similar to those prior to adding the additional capacity. Since new construction is often the last resort to improve the operations of a freeway, strategies that are more cost-effective and utilize existing infrastructure or require minimal expansion are needed to alleviate congestion in the region.

To accomplish this, it is essential to utilize technologies that generate system-wide improvement. The Ada County Highway District is emphasizing technologies involving Active Traffic Management, Adaptive Signals, and other Intelligent Transportation Systems (ITS) (ACHD, 2012). A ramp metering strategy that is predictive, adaptive, and coordinated aligns with the District's focus to identify roadway improvements that plays a role in the development of the Transportation Improvement Plan, which is the regional mid-range transportation plan developed by COMPASS; and the Idaho Transportation Investment Plan, the state-level transportation plan developed by ITD.

I-84 experiences recurring and nonrecurring congestion resulting from bottleneck formations, especially near the Eagle Road interchange. The Eagle Road interchanges two East Bound on-ramps experience some of the heaviest traffic volume along I-84.

Based on these relationships, a predictive feedback on-ramp metering control strategy, that is proactive to the onset of congestion breakdown, was developed. The two Eagle Road EB on-ramps were chosen as test locations for on-ramp metering.

The ramp meter scheme uses a Kalman filter, a recursive forecasting algorithm, to predict traffic density and estimate on-ramp queue lengths and a feedback control logic to adjust the metering rate. The adaptive control scheme works by controlling the traffic density on the mainline while also considering on-ramp queues. Many ramp meter systems in use today have no sophisticated way to handle queues except through queue flushing or queue override.

6.1 Summary of Work

A feedback predictive control scheme was designed for two on-ramps at the Eagle Road interchange on I-84. Radar sensors were installed between the Meridian Road and Eagle Road interchanges from November 6 through November 15 on the EB side of I-84 and November 20 through November 29 in 2013 on the WB side of I-84. The radar sensors collected data for traffic volume, average speeds, occupancy, and vehicle classification. The ramp meter control design was separated into two major research components: prediction and control.

In the first component, the design of dynamic linear models and Kalman filters, used for prediction of the state of traffic, was performed in **R** and MATLAB. The design of the metering control logic algorithm was performed in MATLAB. Finally, these two components were combined into a single integrated algorithmic program.

6.2 Implementation

The KF appears to be a suitable algorithm for this application. It is a simple, recursive data processing algorithm with relatively few parameters to tune. The conversion from ramp metering rate (how many vehicles are allowed to enter the freeway through the ramp per control interval) to the division of signal cycle (the length of green/red phase) that actually controls the meter needs to be decided after many simulations are made in a traffic microsimulation program, such as Vissim (PTV Planung Transport Verkehr AG, 2013). A reasonable minimum cycle is around 4.5 seconds, obtained by a red time of 2.5 seconds and a green time of 2 seconds. Below in Table 6.1 are the FHWA recommended cycle lengths, approximate range of metering rates, and capacities of three ramp scenarios.

Table 6.1. FHWA Recommended Cycle Length, Metering Rates, & Associated Capacity

Flow Control Scheme	No. of Lanes	Cycle Length (sec)	Range of Metering Rates (veh/hr)	Capacity (veh/hr)
One Vehicle Per Green	1	4 – 4.5 Seconds	240 – 900	900
Two Vehicles Per Green	1	6 – 6.5 Seconds	240 – 1200	1100 – 1200
Tandem	2	-	400 – 700	1600 – 1700

Section 5.4.1 presented the tuning of the controls; however, this was for demonstrative purposes only. This is because it is very difficult to analytically determine these values and the impact they will have on the performance of the ramp metering control law (Yasar, Ozbay, & Kachroo, 2006).

6.3 Recommendations for Future Work

With the construction of the new Meridian Interchange, lanes will be increased to four, eliminating the effects of a lane drop. This scenario is advantageous for study because it may have the potential to provide posteriori evidence to the effects of adding an additional lane to a persistent bottleneck. To demonstrate the true capabilities of the prediction and control logic, the code will need to be converted into a language like C++ and converted into a *.dll*. This recommendation is ideal for the Vissim traffic microsimulation software. The prediction algorithms are ready to test in a controller. The modelling, testing, and validation has been extensive and successful enough to warrant the algorithm into a controller for field evaluation.

The METANET traffic model has been shown to accurately represent traffic under a range of conditions (Papageorgiou, Blosseville, & Haj-Salem, 1990a; Papageorgiou et al., 1990b; Wang & Papageorgiou, 2005). This traffic model should be used as the basis for the non-linear equations in the EKF prediction algorithm.

The framework for an integrated predictive and control logic algorithm has been developed. Simulation and optimization needs to be performed to determine the optimal settings for the controller and locations the controllers are being implemented. Yasar et al. (2006) recommends tuning these parameters through a series of multiple microsimulation runs.

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APPENDIX A: MATLAB RAMP METER CONTROL M-FILES

rampmeter_runfile.m

```

% Main File to Run Feedback Ramp Meter Controls with Dual-State KF and
% Decoupled or Coupled Control Schemes
% Data is stochastically Generated Each Run, Based on Observed Data and a
% Moving Average Random Process
% Regulator Gains
    % dcgain: decoupled gain control
    % ccgain: coupled gain for ramp
    % alpha1: distribution factor ramp 1
    % alpha2: distribution factor ramp 2
% Weighting factors
    % w1: weight given to mainline section 1
    % w2: weight given to ramp 1 EB loop on-ramp
    % w3: weight given to mainline section 2
    % w4: weight given to ramp 2 EB on-ramp

clear; clc;

global rhom rhoc1 rhoc2 vf1 vf2 T flow5 ramp1 ramp2 flow4new ...
    alpha1 alpha2 dcgain ccgain w1 w2 w3 w4 L1 L2 RL1 RL2 ...
    G H Q R ii m c meas pplus x_hat resids queue ramp1new ramp2new...
    err RMSEP rampdelay TT uvar1 uvar2

load('rampmeter_wrkspc.mat')

% Input parameters

% c = input('Decoupled D-Mi xcros(1), Coupled C-Mi xcros(2));
c=2;

% Regulator Gains and Weighting Parameters
dcgain=0.90;
ccgain=0.95;
alpha1=0.55;
alpha2=0.45;

w1=.65;
w2=.35;
w3=.65;
w4=.35;

% Known parameters
L1=0.298; % Length of Freeway Section 1 (mi)

```

```

L2=0.391;           % Length of Freeway Section 2 (mi)
RL1=0.38;          % Length of Ramp 1 EB Loop
RL2=0.29;          % Length of Ramp 2 EB On-Ramp
rho_m=110;         % Jam Density (veh/mi)
rho_c1=33;         % Critical Density section 1 (veh/mi)
rho_c2=41;         % Critical Density section 2 (veh/mi)
vf1=73;           % free flow speed section 1 (mi/hr)
vf2=62;           % free flow speed section 2 (mi/hr)
T=5/60;           % Time step

% Kalman Filter design

% State matrix
G = [ 1 0
      0 1];
% State noise
Q = [ 3.7330 0
      0 5.6221];
% Observation matrix
H = [ 1 0
      0 1];
% Observation noise
R = [ 1.1034 0;
      0 2.8308];
% Initial state and error
x0 = [25; 35]; % initial densities
P0 = diag([15 10]);
x_hat = G*x0;
pplus = G*P0*G' + Q;

% Preallocate
m=length(flow5);
X=zeros(m, length(x0));
uvar1=zeros(m, 1);
uvar2=zeros(m, 1);
ramp1new=zeros(m, 1);
ramp2new=zeros(m, 1);
flow4new=zeros(m, 1);
queue=zeros(m, 2);
resids=zeros(m, 2);
err=zeros(m, 2);

% Begin Ramp Meter Prediction & Control Algorithm

meas=[rho1(1) rho2(1)];

for ii=1:m-1;
    % Stochastic Inflow, Ramp 1 and Ramp 2 Volumes
    ramp1new(ii)=ramp1(ii)+(.95+.10.*rand(1)).*(ramp1(ii));
    ramp2new(ii)=ramp2(ii)+(.95+.10.*rand(1)).*(ramp2(ii));
    flow4new(ii)=flow4(ii)+(.95+.10.*rand(1)).*(flow4(ii));

    k=feval('kalman_pred');

```

```

X(ii,:) = k;
input = [k(1), queue(ii, 1), k(2), queue(ii, 2)];
uvar1(ii) = ramp1_meter(ii, input);
uvar2(ii) = ramp2_meter(ii, input);
meas(ii+1,:) = [rho1(ii) + (T)*uvar1(ii), rho2(ii) + (T)*uvar2(ii)];

    if (X(ii, 1) < 0), % Variable Constraints
        X(ii, 1) = 0;
    end
    if (X(ii, 1) > rhom),
        X(ii, 1) = rhom;
    end
    if (X(ii, 2) < 0),
        X(ii+1, 2) = 0;
    end
    if (X(ii, 2) > rhom),
        X(ii, 2) = rhom;
    end
    if (queue(ii, 1) < 0),
        queue(ii, 1) = 0;
    end
    if (queue(ii, 2) < 0),
        queue(ii, 2) = 0;
    end

    queue(ii+1,:) = [(queue(ii, 1) + T*(ramp1new(ii) - uvar1(ii))), ...
                    (queue(ii, 2) + T*(ramp2new(ii) - uvar2(ii)))]];
end

rampdelay = (sum(queue > 0));

% Root mean square prediction error (RMSEP)
RMSEP = sqrt((1/length(flow5)) * sum(abs(resids).^2));

rho1res = meas(:, 1) - X(:, 1); % Store the Residuals
rho2res = meas(:, 2) - X(:, 2); % Store the Residuals
rho1pred = X(:, 1); % Store the Prediction Section 1
rho2pred = X(:, 2); % Store the Prediction Section 2
rho1new = meas(1:end, 1); % Store the new Density Section 1
rho2new = meas(1:end, 2); % Store the new Density Section 2
TT = 1:m; % Time for plots

if c == 1
    c = 'Decoupled Control';
    Decoupled_Plots
    % Writes results to Excel
    D = [dcgain, rampdelay(1), rampdelay(2), w1, w2, w3, w4];
    dlmwrite('Deoupled_Tune.csv', D, '-append', 'delimiter', ',');
end

if c == 2
    c = 'Coupled Control';
    Coupled_Plots

```

```

% Writes results to Excel
D=[dcgain, alpha1, alpha2, rampdelay(1), rampdelay(2), w1, w2, w3, w4];
dlmwrite('Coupled_Tune.csv', D, '-append', 'delimiter', ',');
end

% Plot the results
% Control Variable TI 0-1000
figure
subplot(311)
set(gca, 'FontSize', 9); set(gcf, 'Color', 'White');
plot(TT, ramp1, TT, ramp2, 'r-', 'linewidth', .02);
title(sprintf('w1 =%2.2f, w2 =%2.2f, w3 =%2.2f, w4 =%2.2f, dcgain =%2.2f', ...
    w1, w2, w3, w4, dcgain));
legend('Ramp 1', 'Ramp 2', 'Location', ...
    'Best', 'Orientation', 'horizontal');
ylabel('Flows (veh/hr)');
ylim([0 150])
xlim([0 1000])
hold on
subplot(312)
plot(TT, uvar1, TT, uvar2, 'r-');
set(gca, 'FontSize', 9); set(gcf, 'Color', 'White');
ylabel('Metered flow (veh/hr)');
xlim([0 1000])
ylim([0 150])
hold on
subplot(313)
plot(TT, queue(2:end, 1), '-', TT, queue(2:end, 2), 'r-');
ylabel('Queue Length'); xlabel('Five-minute Intervals');
xlim([0 1000])
ylim([0 20])
set(gca, 'FontSize', 9); set(gcf, 'Color', 'White');
h=suptitle(sprintf('%s', c)); % \n alpha1 = %2.2f, alpha2 = %2.2f, c); % , alpha1, alpha2));
set(h, 'FontSize', 10, 'FontWeight', 'normal');
set(gca, 'FontSize', 9); set(gcf, 'Color', 'White');
hold off

% Control Variable TI 1000-2000
figure
subplot(311);
set(gca, 'FontSize', 8); set(gcf, 'Color', 'White');
plot(TT, ramp1, TT, ramp2, 'r-', 'linewidth', .02);
title(sprintf('Ramp 1 delay =%2.0f, Ramp 2 delay=%2.0f', ...
    rampdelay(1), rampdelay(2)));
legend('Ramp 1', 'Ramp 2', 'Location', ...
    'Best', 'Orientation', 'horizontal');
ylabel('Flows (veh/hr)');
xlim([1000 2000])
ylim([0 150])
hold on
subplot(312)
plot(TT, uvar1, TT, uvar2, 'r-');
set(gca, 'FontSize', 8); set(gcf, 'Color', 'White');

```

```

ylabel('Metered flow (veh/hr)');
xlim([1000 2000])
ylim([0 150])
hold on
subplot(313)
plot(TT, queue(2: end, 1), '- ', TT, queue(2: end, 2), 'r- ');
ylabel('Queue Length'); xlabel('Five-minute Intervals');
xlim([1000 2000])
ylim([0 20])
set(gca, 'FontSize', 8); set(gcf, 'Color', 'White');
h=suptitle(sprintf('%s', c));
set(h, 'FontSize', 10, 'FontWeight', 'normal');
set(gca, 'FontSize', 8); set(gcf, 'Color', 'White');
hold off

% ONE-STEP AHEAD DENSITY SECTION 1
figure
subplot(211)
plot(TT, rho1pred, 'b-o', 'MarkerSize', 4.5);
set(gca, 'FontSize', 10); set(gcf, 'Color', 'White');
title('Density Section 1');
ylabel('veh/mi')
xlim([0 250])
hold on
plot(TT, rho1, 'r: *', 'MarkerSize', 2)
legend('KF', 'Measured', 'Location', 'Best', 'Orientation', 'horizontal');
subplot(212)
plot(TT, rho1pred, 'b-o', 'MarkerSize', 4.5);
ylabel('veh/mi')
hold on
plot(TT, rho1, 'r: *', 'MarkerSize', 2)
h = suptitle('Dual-State KF');
set(h, 'FontSize', 10, 'FontWeight', 'normal');
set(gca, 'FontSize', 10); set(gcf, 'Color', 'White');
xlim([250 500])
xlabel('Five-minute Intervals');
hold off;

figure
subplot(211)
plot(TT, rho1pred, 'b-o', 'MarkerSize', 4.5);
set(gca, 'FontSize', 10); set(gcf, 'Color', 'White');
title('Density Section 1');
ylabel('veh/mi')
xlim([500 750])
hold on
plot(TT, rho1, 'r: *', 'MarkerSize', 2)
legend('KF', 'Measured', 'Location', 'Best', 'Orientation', 'horizontal');
subplot(212)
plot(TT, rho1pred, 'b-o', 'MarkerSize', 4.5);
ylabel('veh/mi')
hold on
plot(TT, rho1, 'r: *', 'MarkerSize', 2)

```

```

h = suprtitle('Dual-State KF');
set(h, 'FontSize', 10, 'FontWeight', 'normal');
set(gca, 'FontSize', 10); set(gcf, 'Color', 'White');
xlim([750 1000])
xlabel('Five-minute Intervals');
hold off;

% ONE-STEP AHEAD DENSITY SECTION 2
figure;
subplot(211)
plot(TT, rho2pred, 'b-o', 'MarkerSize', 4.5);
set(gca, 'FontSize', 10); set(gcf, 'Color', 'White');
title('Density Section 2');
hold on
ylabel('veh/mi')
xlim([1000 1250])
hold on
plot(TT, rho2, 'r:*', 'MarkerSize', 2);
legend('KF', 'Measured', 'Location', 'Best', 'Orientation', 'horizontal');
subplot(212)
plot(TT, rho2pred, 'b-o', 'MarkerSize', 4.5);
hold on
plot(TT, rho2, 'r:*', 'MarkerSize', 2);
h = suprtitle('Dual-State KF');...
set(h, 'FontSize', 10, 'FontWeight', 'normal');
set(gca, 'FontSize', 10); set(gcf, 'Color', 'White');
ylabel('veh/mi')
xlim([1250 1500])
xlabel('Five-minute Intervals');
hold off;

figure;
subplot(211)
plot(TT, rho2pred, 'b-o', 'MarkerSize', 4.5);
set(gca, 'FontSize', 10); set(gcf, 'Color', 'White');
title('Density Section 2');
hold on
ylabel('veh/mi')
xlim([1500 1750])
hold on
plot(TT, rho2, 'r:*', 'MarkerSize', 2);
legend('KF', 'Measured', 'Location', 'Best', 'Orientation', 'horizontal');
subplot(212)
plot(TT, rho2pred, 'b-o', 'MarkerSize', 4.5);
hold on
plot(TT, rho2, 'r:*', 'MarkerSize', 2);
h = suprtitle('Dual-State KF');...
set(h, 'FontSize', 10, 'FontWeight', 'normal');
set(gca, 'FontSize', 10); set(gcf, 'Color', 'White');
ylabel('veh/mi')
xlim([1750 2000])
xlabel('Five-minute Intervals');
hold off;

```



```

% Residuals
figure
subplot(211)
plot(TT, resid(:, 1), '-');
set(gca, 'FontSize', 10); set(gcf, 'Color', 'White');
title(sprintf('RMSEP Sec 1 = %2.2f, Sec 2 = %2.2f', ...
    RMSEP(1), RMSEP(2)));
hold on
ylabel('Section 1');
xlim([0 2000])
hold on
subplot(212)
plot(TT, resid(:, 2), '-')
xlabel('Five-minute Intervals'); ylabel('Section 2')
set(gca, 'FontSize', 9); set(gcf, 'Color', 'White');
h=suptitle('Dual-State KF Residuals');
set(h, 'FontSize', 10, 'FontWeight', 'normal');
set(gca, 'FontSize', 10); set(gcf, 'Color', 'White');
xlim([0 2000])

```

ramp1_meter.m

```

function u1 = ramp1_meter(t,x)
% Ramp 1 Metered Outflow

global c rhoc1 rhoc2 rhom vf1 vf2 flow4new w1 w2 w3 w4 L1 L2 RL1 RL2...
    dcgain ccgain alpha1 T queue ramp1new ramp2new err

q1out=vf1*x(1)*(1-(x(1)/rhom)); % flow out section 1
q2out=vf2*x(3)*(1-(x(3)/rhom)); % flow out section 2

% Decoupled Coordinated Control
% D-MIXCROS
if c == 1
    F1=w1*sign((x(1)-rhoc1)+(T/L1)*(-q1out+flow4new(t)))+...
        w2*(x(2)+(T/RL1)*ramp1new(t)); % (veh/mi)
    G1=(sign(x(1)-rhoc1)*w1*(T/L1)-w2*(T/RL1))^-1; % (mi/hr)
    e = w1*abs(x(1)-rhoc1) + w2*abs(x(2)); % (veh/mi)
    err(t,1)=e; % store error
    u1 = max(0, (-F1-dcgain*e)*G1); % (veh/hr)

% Coupled Distributed Coordinated Control
% C-MIXCROS
elseif c == 2
    F1=w1*sign((x(1)-rhoc1)+(T/L1)*(-q1out+flow4new(t)))+...
        w2*(x(2)+(T/RL1)*ramp1new(t));
    F2=w3*sign((x(3)-rhoc2)+(T/L2)*(-q2out+q1out))+...
        w4*(x(4)+T/RL2*ramp2new(t));
    F=F1+F2; % (veh/mi)
    G1=(sign(x(1)-rhoc1)*w1*(T/L1)-w2*(T/RL1))^-1; % (mi/hr)
    e = w1*abs(x(1)-rhoc1)+w2*abs(x(2))+...
        w3*abs(x(3)-rhoc2)+w4*abs(x(4)); % (veh/mi)
    err(t,2)=e; % store error
    u1 = max(0, (G1*(alpha1*(-F-ccgain*e)))); % (veh/hr)

    if queue(t,1) <= 0
        if ramp1new(t) < u1
            u1 = ramp1new(t);
        end
    end
    if x(1) > rhom
        u1 = 0;
    end

% Ramp Meter Off
else
    u1=ramp1new(t);
end

```

ramp2_meter.m

```

function u2 = ramp2_meter(t,x)
% Ramp 2 Metered Outflow

global c rhoc1 rhoc2 rhom vf1 vf2 flow4new w1 w2 w3 w4 L1 L2 RL1 RL2...
    dcgain ccgain alpha2 T queue ramp1new ramp2new err

q1out=vf1*x(1)*(1-(x(1)/rhom)); % flow out section 1
q2out=vf2*x(3)*(1-(x(3)/rhom)); % flow out section 2

% Decoupled Coordinated Control
% D-MI XCROS
if c == 1
    F2=w3*sign((x(3)-rhoc2)+(T/L2)*(-q2out+q1out))+...
        w4*(x(4)+T/RL2*ramp2new(t)); % (veh/mi)
    G2=(sign(x(3)-rhoc2)*w3*(T/L2)-w4*(T/RL2))^-1; % (mi/hr)
    e = w3*abs(x(3)-rhoc2) + w4*abs(x(2)); % (veh/mi)
    err(t,2)=e; % store error
    u2 = max(0, (-F2-dcgain*e)*G2); % (veh/hr)

% Coupled Distributed Coordinated Control
% C-MI XCROS
elseif c == 2
    F1=w1*sign((x(1)-rhoc1)+(T/L1)*(-q1out+flow4new(t)))+...
        w2*(x(2)+T/RL1*ramp1new(t));
    F2=w3*sign((x(3)-rhoc2)+(T/L2)*(-q2out+q1out))+...
        w4*(x(4)+T/RL2*ramp2new(t));
    F=F1+F2; % (veh/mi)
    G2=(sign(x(3)-rhoc2)*w3*(T/L2)-w4*(T/RL2))^-1; % (mi/hr)
    e = (w1*abs(x(1)-rhoc1)+w2*abs(x(2))+...
        w3*abs(x(3)-rhoc2)+w4*abs(x(4))); % (veh/mi)
    err(t,2)=e; % store error
    u2 = max(0, (G2*(alpha2*(-F-ccgain*e)))); % (veh/hr)

    if queue(t,2) <= 0
        if ramp2new(t) < u2
            u2 = ramp2new(t);
        end
    end
    if x(3) > rhom
        u2 = 0;
    end

% Ramp Meter Off
else
    u2=ramp2new(t);
end

```

kalman_pred.m

```

function [predrho] = kalman_pred
% Kalman Filter for density prediction sections 1 & 2 using measured
% flows & speeds from EBS5 and EBS6

global meas F H Q R ii pplus x_hat resids

% Filter
xpred = G*x_hat; % eq (3.5)
predrho=[xpred(1); xpred(2)]; % store predictions
ppred = G*pplus* G ' +Q; % eq (3.6)
y_hat = meas(ii, :) ' - F*xpred; % eq (3.7) Innovation
resids(ii, :)=y_hat'; % store residual
K = (ppred*F')/(F*ppred*F' + R); % eq (3.9) Kalman Gain
x_hat = xpred + K*y_hat; % eq (3.10)
pplus =(eye(2)-K*F)*ppred*(eye(2)-K*F)' + K*R*K'; % eq (5.1) (Joseph form)

```

APPENDIX B: EXTENDED KALMAN FILTER MATLAB M-FILE

EKF.m

```

% Extended Kalman Filter for Density Predictions

clear; clc;

global xhat Q R P flow5 vf1 vf2 L1 L2 rhom1 rhom2
load('rampmeter_wrkspc.mat')

% Known Parameters
L1=0.298;           % length of freeway section 1 (mi)
L2=0.391;           % length of freeway section 2 (mi)
rhom1=85;           % Jam Density section 1 (veh/mi)
rhom2=105;          % Jam Density section 2 (veh/mi)
rhoc1=32;           % Critical Density section 1 (veh/mi)
rhoc2=24;           % Critical Density section 2 (veh/mi)
vf1=72;             % free flow speed section 1(mi/hr)
vf2=65;             % free flow speed section 2 (mi/hr)
T=5/60;             % Time-step
TT=1:2151;          % Time for plots

% Preallocate
rho1pred=zeros(length(flow5), 1);
queue1=zeros(length(flow5), 1);
rho2pred=zeros(length(flow5), 1);
queue2=zeros(length(flow5), 1);
r1=zeros(length(flow5), 1);
r2=zeros(length(flow5), 1);
res=zeros(length(flow5), 2);
measured=zeros(length(flow5), 2);
residuals = zeros(length(flow5), 2);

% Filter Design

% Initialization
xhat = [45; 58];
P = [80 .90
     .02 19];
Q = [3.2 .020^2
     1 25];
R = [5 3.1
     1.9 .90];

```

```

% Jacobian for the state
F= [(T*(vf1*(xhat(1)/rhom1-1)+(vf1*xhat(1)/rhom1))/L1, 0;
    -(T*(vf1*(xhat(1)/rhom1-1)+(vf1*xhat(1)/rhom1))/L2, ...
    (T*(vf2*(xhat(2)/rhom2-1)+(vf2*xhat(2)/rhom2))/L2];

% Jacobian for the measurement equations
H = [1 0;
     0 1];

% Begin Algorithm
for ii=1:2151

% Measurement
meas=[rho1(ii); rho2(ii)];
measured(ii,:)=meas;

% Propagate the State Matrix
xhat=F*xhat;

    % store the predictions
    rho1pred(ii)=xhat(1);
    rho2pred(ii)=xhat(2);

% Propagate the Covariance Matrix
P = F*P*F' + Q;

% Calculate the Kalman gain
K = P*H'/(H*P*H' + R);

% Calculate the Measurement Residual
yhat = [xhat(1); xhat(2)];
resid = meas - yhat;
residuals(ii,:) = resid(:,1)';

% Update the State and Covariance Estimates
xhat = xhat + K*resid;
P = (eye(size(K,1))-K*H)*P;
end

RMSEP=sqrt((1/length(flow4))*sum(abs(residuals).^2));
rho1res=measured(:,1)-rho1;
rho2res=measured(:,2)-rho2;

% Plot the results
% One-Step Ahead Predictions Density Section 1
figure
subplot(211)
set(gca, 'FontSize', 9); set(gcf, 'Color', 'White');
plot(TT, rho1pred, 'r-*', 'MarkerSize', 4);
title('Density Section 1');
ylabel('veh/mi')
hold on
plot(TT, rho1, 'b:o', 'MarkerSize', 3);

```

```

legend('EKF', 'Measured', 'Location', 'Northeast');
set(gca, 'FontSize', 9); set(gcf, 'Color', 'White');
xlim([0 500])
ylim([0 40])
subplot(212)
title('Density Section 1');
plot(TT, rho1pred, 'r-*', 'MarkerSize', 4);
hold on
plot(TT, rho1, 'b:o', 'MarkerSize', 3);
set(gca, 'FontSize', 9); set(gcf, 'Color', 'White');
xlim([500 1000])
h = supitle('Extended Kalman Filter');
set(h, 'FontSize', 11, 'FontWeight', 'normal');
xlabel('Five-minute Intervals'); ylabel('veh/mi')
hold off

figure
subplot(211)
set(gca, 'FontSize', 9); set(gcf, 'Color', 'White');
plot(TT, rho1pred, 'r-*', 'MarkerSize', 4);
title('Density Section 1');
ylabel('veh/mi')
hold on
plot(TT, rho1, 'b:o', 'MarkerSize', 3);
legend('EKF', 'Measured', 'Location', 'Northeast');
set(gca, 'FontSize', 9); set(gcf, 'Color', 'White');
xlim([1000 1500])
ylim([0 40])
subplot(212)
title('Density Section 1');
plot(TT, rho1pred, 'r-*', 'MarkerSize', 4);
hold on
plot(TT, rho1, 'b:o', 'MarkerSize', 3);
set(gca, 'FontSize', 9); set(gcf, 'Color', 'White');
xlim([1500 2000])
ylim([0 40])
h = supitle('Extended Kalman Filter');
set(h, 'FontSize', 11, 'FontWeight', 'normal');
xlabel('Five-minute Intervals'); ylabel('veh/mi')
hold off

% One-Step Ahead Predictions Density Section 2
figure;
subplot(211)
plot(TT, rho2pred, 'r-*', 'MarkerSize', 4);
set(gca, 'FontSize', 9); set(gcf, 'Color', 'White');
title('Density Section 2');
ylabel('veh/mi')
xlim([0 500])
ylim([0 40])
hold on
plot(TT, rho2, 'b:o', 'MarkerSize', 3);
legend('EKF', 'Measured', 'Location', 'Best');

```

```

subplot(212)
plot(TT, rho2pred, 'r- *', 'MarkerSize', 4);
set(gca, 'FontSize', 9); set(gcf, 'Color', 'White');
hold on
plot(TT, rho2, 'b: o', 'MarkerSize', 3);
xlim([500 1000])
ylim([0 30])
h = supitle('Extended Kalman Filter');
set(h, 'FontSize', 11, 'FontWeight', 'normal');
xlabel('Five-minute Intervals'); ylabel('veh/mi')
hold off;

figure;
subplot(211)
plot(TT, rho2pred, 'r- *', 'MarkerSize', 4);
set(gca, 'FontSize', 9); set(gcf, 'Color', 'White');
title('Density Section 2');
ylabel('veh/mi')
xlim([1000 1500])
ylim([0 40])
hold on
plot(TT, rho2, 'b: o', 'MarkerSize', 3);
legend('EKF', 'Measured', 'Location', 'Best');
subplot(212)
plot(TT, rho2pred, 'r- *', 'MarkerSize', 4);
set(gca, 'FontSize', 9); set(gcf, 'Color', 'White');
hold on
plot(TT, rho2, 'b: o', 'MarkerSize', 3);
xlim([1500 2000])
ylim([0 30])
h = supitle('Extended Kalman Filter');
set(h, 'FontSize', 11, 'FontWeight', 'normal');
xlabel('Five-minute Intervals'); ylabel('veh/mi')
hold off;

% Residuals
figure
subplot(211)
plot(1:length(flow4), residuals(:, 1), '- ')
ylabel('Section 1');
xlim([0 2000])
ylim([-25 25])
title(sprintf('RMSEP Section 1 = %.2f, Section 2 = %.2f', ...
    RMSEP(1), RMSEP(2)));
set(gca, 'FontSize', 9); set(gcf, 'Color', 'White');
subplot(212)
plot(1:length(speed4), residuals(:, 2), '- ')
set(gca, 'FontSize', 9); set(gcf, 'Color', 'White');
xlabel('Five-minute Intervals'); ylabel('Section 2')
h=supitle('EKF Residuals');
set(h, 'FontSize', 10, 'FontWeight', 'normal');
ylim([-25 25])
xlim([0 2000])

```


APPENDIX C: R CODE: TRAFFIC VOLUME DLMS & KF

Section_4.2.R

```

## Code for Thesis Chapter 4.2 DLM & KF ##
## 4.2 R Model Specification and Parameter Estimation: Traffic Volume ##

library(dlm)
library(forecast)
library(numDeriv)

## 4.2.3 Random Walk ##
buildvol <- function(theta) {
  dlmModPoly(order = 1, dV = theta[1], dW = theta[2])}

fitvol <- dlmMLE(vol, parm = c(0, 150), buildvol, hessian=T, lower = rep(1e-4, 2))
## MLE of Unknown parameters
modvol <- buildvol(fitvol$par) ## Fitted model
fitvol$convergence ## Check Convergence
drop(W(modvol))[1] ## System Variance
drop(V(modvol)) ## Observation Variance

hs <- hessian(function(x) dlmLL(vol, buildvol(x)), fitvol$par)
all(eigen(hs, only.values = TRUE)$values > 0) ## Positive Definite?
aVar <- solve(hs) ## Asymptotic Variance/Covariance Matrix
sqrt(diag(aVar)) ## Standard Errors

## Kalman Filter ##
volFilt <- dlmFilter(vol, modvol) # Kalman Filter
RW_res <- residuals(volFilt, sd=FALSE)

## Plots Random Walk ##
x11(width=6, height=6.25, pointsize=12)
par(mar=c(5, 4, 4, 1, 2) - .6, mex=0.9)
par(mfrow=c(4, 1))
plot.ts(vol, xlab="Five-Minute Intervals", ylab="vehicle volume", type='o',
        ylim=c(0, 120), xaxs="i")
lines(window(volFilt$f, start = start(vol) +
c(1, 0)), lty=4, pch=4, lwd=.25, type="o", col="red")
plot.ts(vol, xlab="Five-Minute Intervals", type='o', ylab="vehicle volume",
        xlim=c(0, 100), ylim=c(0, 110), xaxs="i")
lines(window(volFilt$f, start = start(vol) +
c(0, 0)), lty=4, pch=4, lwd=1.5, type="o", col="red")
plot.ts(vol, xlab="Five-Minute Intervals", type='o', ylab="vehicle volume",
        xlim=c(100, 200), ylim=c(0, 110), xaxs="i")

```

```

lines(window(volFilt$f, start = start(vol)+
c(0,0)), lty=4, pch=4, lwd=1.5, type="o", col="red")
plot.ts(vol, type='o', ylab="vehicle volume", xlab="Five-Minute Intervals",
        xlim=c(900,1000), ylim=c(0,110), xaxs="i")
lines(window(volFilt$f, start = start(vol) +
c(0,0)), lty=4, pch=4, lwd=1.5, type="o", col="red")
legend("bottomright", leg = c("Measured", "One-Step Ahead Forecasts"),
       cex = 0.9, lty = c(1, 2), col = c("black", "red"),
       pch=c(1,4), bty = "y", horiz = T)
#####

## 4.2.4 RW with Seasonal Component ##
buildvolseas <- function(theta) {
  dlModPoly(order = 1, dV = theta[1], dW = theta[2])+
  dlModSeas(12, dV=10)}

fitvolseas <- dlMLE(vol, parm = c(0, 150), buildvolseas, hessian=T, lower = rep(1e-4, 2))
## MLE of Unknown parameters
modvolseas <- buildvolseas(fitvolseas$par) ## Fitted model
fitvol$convergence ## Check Convergence
drop(W(modvol))[1] ## System Variance
drop(V(modvol)) ## Observation Variance
hs <- hessian(function(x) dlMLL(vol, buildvolseas(x)), fitvolseas$par)
all(eigen(hs, only.values = TRUE)$values > 0) ## Positive Definite?
aVar <- solve(hs) ## Asymptotic Variance/Covariance Matrix
sqrt(diag(aVar)) ## Standard Errors

## Kalman Filter ##
volFiltseas <- dlmFilter(vol, modvolseas) # Kalman Filter
seas_res <- residuals(volFiltseas, sd=FALSE)

## Plots Random Walk with Seasonal ##
x11(width=6, height=6.25, pointsize=12)
par(mar=c(5, 4.4, 1, 2) - .6, mex=0.9)
par(mfrow=c(4, 1))
plot.ts(vol, xlab="Five-Minute Intervals", ylab="vehicle volume", type='o',
        ylim=c(0,120), xaxs="i")
lines(window(volFiltseas$f, start = start(vol)+ c(1,0)),
      lty=4, pch=4, lwd=.25, type="o", col="red")
plot.ts(vol, xlab="Five-Minute Intervals", type='o', ylab="vehicle volume",
        xlim=c(0,100), ylim=c(0,110), xaxs="i")
lines(window(volFiltseas$f, start = start(vol)+ c(0,0)),
      lty=4, pch=4, lwd=1.5, type="o", col="red")
plot.ts(vol, xlab="Five-Minute Intervals", type='o', ylab="vehicle volume",
        xlim=c(100,200), ylim=c(0,110), xaxs="i")
lines(window(volFiltseas$f, start = start(vol)+ c(0,0)),
      lty=4, pch=4, lwd=1.5, type="o", col="red")
plot.ts(vol, type='o', ylab="vehicle volume", xlab="Five-Minute Intervals",
        xlim=c(900,1000), ylim=c(0,110), xaxs="i")
lines(window(volFiltseas$f, start = start(vol) + c(0,0)),
      lty=4, pch=4, lwd=1.5, type="o", col="red")
legend("bottomright", leg = c("Measured", "One-Step Ahead Forecasts"),
       cex = 0.9, lty = c(1, 2), col = c("black", "red"),

```

```

    pch=c(1,4), bty = "y", horiz = T)
#####

## 4.2.5 Random Walk with Trig Seasonal ##
vol=ts(vol)
buildvoltrig <- function(theta) {
  dlModPoly(order = 1, dV = theta[1], dW = theta[2])+
  dlModTrig(s=12, q=6)}

fitvoltrig <- dlMLE(vol, parm = c(0,150), buildvoltrig, hessian=T, lower = rep(1e-4, 2))
## MLE of Unknown parameters
modvoltrig <- buildvoltrig(fitvoltrig$par) ## Fitted model
fitvol$convergence ## Check Convergence
drop(W(modvol))[1] ## System Variance
drop(V(modvol)) ## Observation Variance
hs <- hessian(function(x) dlMLL(vol, buildvoltrig(x)), fitvoltrig$par)
all(eigen(hs, only.values = TRUE)$values > 0) ## Positive Definite?
aVar <- solve(hs) ## Asymptotic Variance/Covariance Matrix
sqrt(diag(aVar)) ## Standard Errors

### Kalman Filter ###
volFilttrig <- dlFilter(vol, modvoltrig) # Kalman Filter
trig_res <- residuals(volFiltseas, sd=FALSE)

## Plots, Random Walk with Trig Seasonal ##
x11(width=6, height=6.25, pointsize=12)
par(mar=c(5, 4.4, 1, 2) - .6, mex=0.9)
par(mfrow=c(4, 1))
plot.ts(vol, xlab="Five-Minute Intervals", ylab="vehicle volume", type='o',
        ylim=c(0, 120), xaxs="i")
lines(window(volFilttrig$f, start = start(vol) + c(1, 0)),
      lty=4, pch=4, lwd=.25, type="o", col="red")
plot.ts(vol, xlab="Five-Minute Intervals", type='o', ylab="vehicle volume",
        xlim=c(0, 100), ylim=c(0, 110), xaxs="i")
lines(window(volFilttrig$f, start = start(vol) + c(0, 0)),
      lty=4, pch=4, lwd=1.5, type="o", col="red")
plot.ts(vol, xlab="Five-Minute Intervals", type='o', ylab="vehicle volume",
        xlim=c(100, 200), ylim=c(0, 110), xaxs="i")
lines(window(volFilttrig$f, start = start(vol) + c(0, 0)),
      lty=4, pch=4, lwd=1.5, type="o", col="red")
plot.ts(vol, type='o', ylab="vehicle volume", xlab="Five-Minute Intervals",
        xlim=c(900, 1000), ylim=c(0, 120), xaxs="i")
lines(window(volFilttrig$f, start = start(vol) + c(0, 0)),
      lty=4, pch=4, lwd=1.5, type="o", col="red")
legend("topright", leg = c("Measured", "One-Step Ahead Forecasts"),
      cex = 0.9, lty = c(1, 2), col = c("black", "red"),
      pch=c(1, 4), bty = "y", horiz = T)
#####

```

```
## Test Statistics ##

## RW Model ##
## Mean Average Deviation (MAD)
mean(c(abs(volFilt$f-vol)), na.rm=TRUE )
## Root Mean Square Error Of Prediction (RMSEP)
sqrt(1/length(vol)*sum(c(abs((volFilt$f-vol)^2)), na.rm=TRUE))

## Seasonal Based Model ##
## Mean Average Deviation (MAD)
mean(c(abs(volFiltseas$f-vol)), na.rm=TRUE )
## Root Mean Square Error Of Prediction (RMSEP)
sqrt(1/length(vol)*sum(c(abs((volFiltseas$f-vol)^2)), na.rm=TRUE))

## RW with Fourier-form Model ##
## Mean Average Deviation (MAD)
mean(c(abs(volFilttrig$f-vol)), na.rm=TRUE )
## Root Mean Square Error Of Prediction (RMSEP)
sqrt(1/length(vol)*sum(c(abs((volFilttrig$f-vol)^2)), na.rm=TRUE))
```

APPENDIX D: R CODE: TRAFFIC SPEEDS DLMS & KF

Section_4.3.R

```

## Code for Thesis Chapter 4.3 DLM & KF ##
## 4.3 R Model Specification and Parameter Estimation: Traffic Speeds ##

library(dlm)
library(forecast)
library(numDeriv)

Thesis_4.3 <- read.csv("C:/Users/Brian/SkyDrive/Forecasting Papers/SSwithKFfor
forecast/Kalman/Data/Thesis_4.3.csv")
spd1 <- ts(Thesis_4.3[,c(2)])

## 4.3.3 Random Walk ##
buildspd1 <- function(theta) {
  dlmModPoly(order = 1, dV = theta[1], dW = theta[2])}

fitspd1 <- dlmMLE(spd1, parm = c(.2, 50, 75), buildspd1, hessian=T, lower = rep(1e-6))
## MLE of Unknown parameters

modspd1 <- buildspd1(fitspd1$par) ## Fitted model
fitspd1$convergence ## Check Convergence
drop(W(modspd1))[1] ## System Variance
drop(V(modspd1)) ## Observation Variance
hs <- hessian(function(x) dlmLL(spd1, buildspd1(x)), fitspd1$par)
all(eigen(hs, only.values = TRUE)$values > 0) ## Positive Definite?
aVar <- solve(hs) ## Asymptotic Variance/Covariance Matrix
sqrt(diag(aVar)) ## Standard Errors

### Kalman Filter ###
spd1filt <- dlmFilter(spd1, modspd1) # Kalman Filter

## 4.3.3 Plots Random Walk##
x11(width=6, height=6.25, pointsize=12)
par(mar=c(5, 4.4, 1, 2)-.6, mex=0.9)
## 4.3.3 Plots Random Walk##
x11(width=6, height=6.25, pointsize=12)
par(mar=c(5, 4.4, 1, 2)-.6, mex=0.9) par(mfrow=c(4, 1))
plot.ts(spd1, xlab="Five-Minute Intervals", ylab="mph", type='o', ylim=c(10, 90), xaxs="i")
lines(window(spd1filt$f, start = start(spd1)+ c(1, 0)),
      lty=4, pch=4, lwd=.25, type="o", col="red")
plot.ts(spd1, xlab="Five-Minute Intervals", type='o', ylab="mph", xlim=c(0, 100),
        ylim=c(10, 90), xaxs="i")
lines(window(spd1filt$f, start = start(spd1)+c(0, 0)),

```

```

lty=4, pch=4, lwd=1.5, type="o", col="red")
plot.ts(spd1, xlab="Five-Minute Intervals", type='o', ylab="mph",
        xlim=c(100, 200), ylim=c(20, 90), xaxs="i")
lines(window(spd1Filt$f, start = start(spd1) + c(0, 0)),
      lty=4, pch=4, lwd=1.5, type="o", col="red")
plot.ts(spd1, type='o', ylab="mph", xlab="Five-Minute Intervals",
        xlim=c(600, 720), ylim=c(20, 90), xaxs="i")
lines(window(spd1Filt$f, start = start(spd1) + c(0, 0)),
      lty=4, pch=4, lwd=1.5, type="o", col="red")
legend("topright", leg = c("Measured", "One-Step Ahead Forecasts"),
      cex = 0.9, lty = c(1, 2), col = c("black", "red"),
      pch=c(1, 4), bty = "y", horiz = T)
#####

spd2 <- ts(spd1, frequency=12) ## changed to one-hour observation periods
## Speed Model w/ Seasonal Component ##
buildspdseas <- function(theta) {
  dlModPoly(order = 1, dV = theta[1], dW = theta[2]) +
  dlModSeas(12)}

fitspdseas <- dlMLE(spd2, parm = c(.1, 25), buildspdseas, hessian=T,
  lower = rep(1e-6)) ## MLE of Unknown parameters
modspdseas <- buildspdseas(fitspdseas$par) ## Fitted model
fitspd2$convergence ## Check Convergence

drop(W(modspdseas))[1] ## System Variance
drop(V(modspdseas)) ## Observation Variance
hs <- hessian(function(x) dlMLL(spd2, buildspdseas(x)), fitspdseas$par)
all(eigen(hs, only.values = TRUE)$values > 0) ## Positive Definite?
aVar <- solve(hs) ## Asymptotic Variance/Covariance Matrix
sqrt(diag(aVar)) ## Standard Errors

### Kalman Filter ###
spdseasFilt <- dlmFilter(spd2, modspdseas) # Kalman Filter

## 4.3.4 Plots Random Walk with Seasonal ##
x11(width=6, height=6.25, pointsize=12)
par(mar=c(5, 4, 4, 1, 2) - .6, mex=0.9)
par(mfrow=c(4, 1))
plot.ts(spd2, xlab="One-Hour Intervals", ylab="mph", type='o',
        ylim=c(10, 90), xaxs="i")
lines(window(spdseasFilt$f, start = start(spd2) + c(1, 0)),
      lty=4, pch=4, lwd=.25, type="o", col="red")
plot.ts(spd2, xlab="One-Hour Intervals", type='o', ylab="mph",
        xlim=c(0, 20), ylim=c(10, 90), xaxs="i")
lines(window(spdseasFilt$f, start = start(spd2) + c(0, 0)),
      lty=4, pch=4, lwd=1.5, type="o", col="red")
plot.ts(spd2, xlab="One-Hour Intervals", type='o', ylab="mph",
        xlim=c(20, 40), ylim=c(20, 90), xaxs="i")
lines(window(spdseasFilt$f, start = start(spd2) + c(0, 0)),
      lty=4, pch=4, lwd=1.5, type="o", col="red")
plot.ts(spd2, type='o', ylab="mph", xlab="One-Hour Intervals",
        xlim=c(40, 65), ylim=c(20, 90), xaxs="i")
lines(window(spdseasFilt$f, start = start(spd2) +

```

```

c(0, 0), lty=4, pch=4, lwd=1.5, type="o", col="red")
legend("topright", leg = c("Measured", "One-Hour Intervals"),
      cex = 0.9, lty = c(1, 2), col = c("black", "red"),
      pch=c(1, 4), bty = "y", horiz = T)
#####

## 4.3.5 Random Walk Fourier-form ##
buildspdtrig <- function(theta) {
  dlModPoly(order = 1, dV = theta[1], dW = theta[2]) +
  dlModTrig(s=12, q=6)}

fitspdtrig <- dlMLE(spd1, parm = c(.1, 25), buildspdtrig, hessian=T, lower = rep(1e-6))
## MLE of Unknown parameters
modspdtrig <- buildspdtrig(fitspdtrig$par) ## Fitted model
fitspd1$convergence ## Check Convergence
drop(W(modspdtrig))[1] ## System Variance
drop(V(modspdtrig)) ## Observation Variance
hs <- hessian(function(x) dlMLL(spd1, buildspdtrig(x)), fitspdtrig$par)
all(eigen(hs, only.values = TRUE)$values > 0) ## Positive Definite?
aVar <- solve(hs) ## Asymptotic Variance/Covariance Matrix
sqrt(diag(aVar)) ## Standard Errors

### Kalman Filter ###
spdtrigFilt <- dlFilter(spd2, modspdtrig) # Kalman Filter

## 4.3.5 Plots Random Walk Fourier-form ##
x11(width=6, height=6.25, pointsize=12)
par(mar=c(5, 4, 4, 1, 2) - .6, mex=0.9)
par(mfrow=c(4, 1))
plot.ts(spd2, xlab="One-Hour Intervals", ylab="mph", type='o',
        ylim=c(10, 90), xaxs="i")
lines(window(spdtrigFilt$f, start = start(spd1) + c(1, 0)),
      lty=4, pch=4, lwd=.25, type="o", col="red")
plot.ts(spd2, xlab="One-Hour Intervals", type='o', ylab="mph",
        xlim=c(0, 20), ylim=c(10, 90), xaxs="i")
lines(window(spdtrigFilt$f, start = start(spd1) + c(0, 0)),
      lty=4, pch=4, lwd=1.5, type="o", col="red")
plot.ts(spd2, xlab="One-Hour Intervals", type='o', ylab="mph",
        xlim=c(20, 40), ylim=c(20, 90), xaxs="i")
lines(window(spdtrigFilt$f, start = start(spd1) + c(0, 0)),
      lty=4, pch=4, lwd=1.5, type="o", col="red")
plot.ts(spd2, type='o', ylab="mph", xlab="One-Hour Intervals",
        xlim=c(40, 65), ylim=c(20, 90), xaxs="i")
lines(window(spdtrigFilt$f, start = start(spd1) + c(0, 0)),
      lty=4, pch=4, lwd=1.5, type="o", col="red")
legend("topright", leg = c("Measured", "One-Step Ahead Forecasts"),
      cex = 0.9, lty = c(1, 2), col = c("black", "red"),
      pch=c(1, 4), bty = "y", horiz = T)
#####

```

```

## Test Statistics ##

## RW Model ##
## Mean Average Deviation (MAD)
mean(c(abs(spd1Filt$f- spd1)), na.rm=TRUE )
## Root Mean Square Error Of Prediction (RMSEP)
sqrt(1/length(spd1)*sum(c(abs((spd1Filt$f- spd1)^2)), na.rm=TRUE))

## Seasonal Based Model ##
## Mean Average Deviation (MAD)
mean(c(abs(spdseasFilt$f- spd2)), na.rm=TRUE )
## Root Mean Square Error Of Prediction (RMSEP)
sqrt(1/length(spd2)*sum(c(abs((spdseasFilt$f- spd2)^2)), na.rm=TRUE))

## RW with Fourier-form Model ##
## Mean Average Deviation (MAD)
mean(c(abs(spdtrigFilt$f- spd2)), na.rm=TRUE )
## Root Mean Square Error Of Prediction (RMSEP)
sqrt(1/length(spd2)*sum(c(abs((spdtrigFilt$f- spd2)^2)), na.rm=TRUE))

```


APPENDIX E: PLOTS

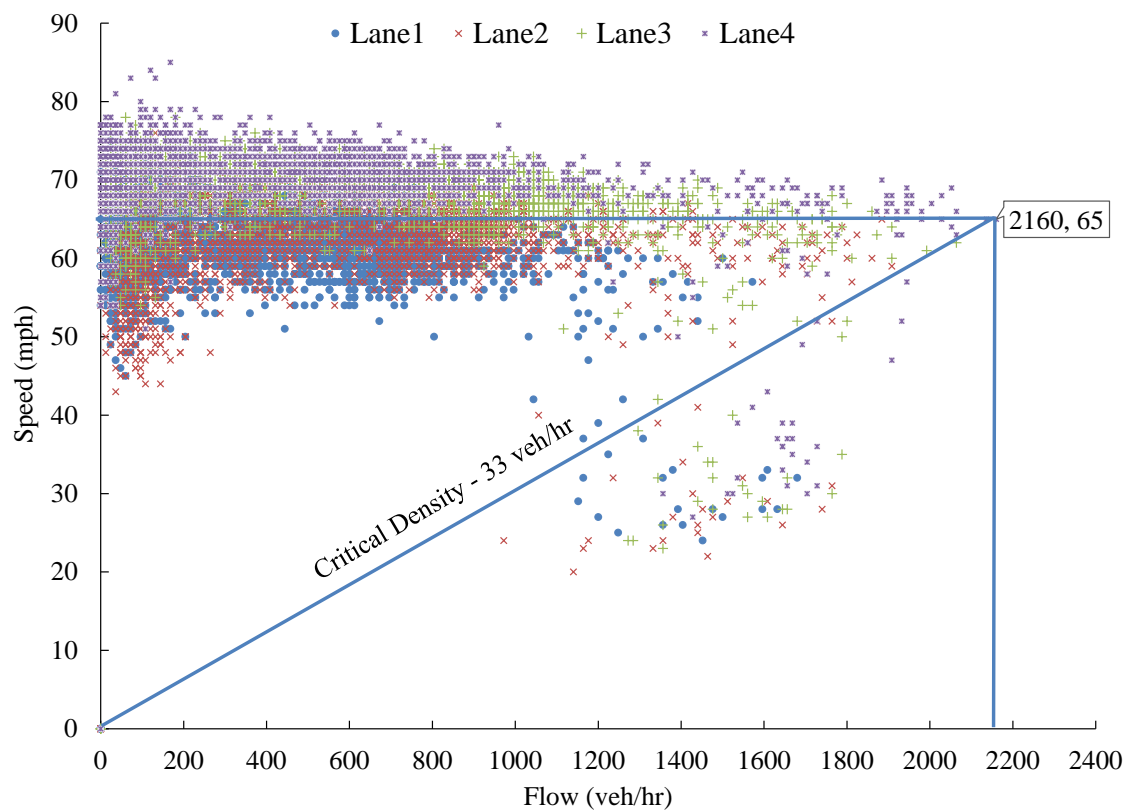


Figure 42. Section 1 Speed-Flow Diagram Displaying Critical Density (ρ_{oc1})

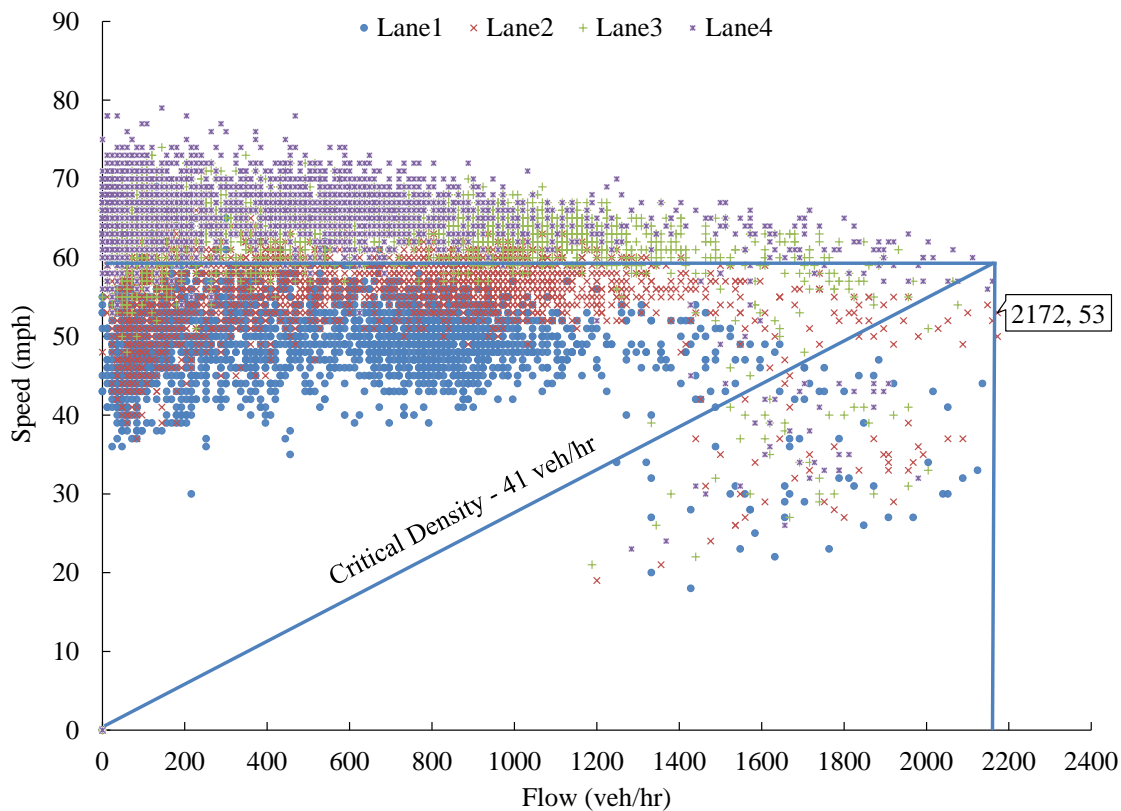


Figure 43. Section 2 Speed-Flow Diagram Displaying Critical Density (ρ_{oc2})